



Case Study 03

Basic data

Title of Research Teaching Unit	Applications of vectors in everyday life (Maps)
Unit created by	BORG Deutsch-Wagram (AT)
Based on Concept	Travelling on a map with vectors
Concept created by	Škola da Vinci (CZ)
Date of teaching	20.11.2023
Number of students in class	24
Number of migrant students	4

1. Unit Description of Research Teaching Unit

Title	Applications of vectors in everyday life (Maps)
Age of students/grade	15 years
Length of the unit	1 x 50 minutes
Mathematical topic	Vectors
Prerequisite knowledge	<ul style="list-style-type: none">• Properties of vectors in the plane• Lines in the plane• Parametric representation of a line
Materials needed	<ul style="list-style-type: none">• Work sheet “Calculations with vectors in Austria” (1 copy per student)• “Map of Austria” (one per team, or alternatively printed on the back of the work sheet for each student)
Objectives	<ul style="list-style-type: none">• Applying vectors in real-world problems• Using coordinates in a real-life map• Modelling using vectors
Notes on work with migrant students	<ul style="list-style-type: none">• As a possible extension, each team could create their own tasks, using suitable maps and city names from students’ background countries

Lesson plan:

The lesson is planned for the end of the topic “vectors in the plane” and “lines in the plane”.

Time	Method	Content and comments
ca. 10 min	Exposition and Teacher-student-communication	The teacher motivates the students for the topic of the lesson and describes the importance of percentages in everyday life. The teacher, together with the students, repeats the main concepts and properties of vectors (reading vector coordinates off a graphical representation, norm of a vector, parametric representation of a line). This could also be written down on the blackboard
ca. 25 min	Partner work	The students solve the tasks of the work sheet in partner work (2 students). The teacher supports the teams if required.
ca. 15 min	Teacher-student-communication	Comparison of the results

Calculations with vectors in Austria – Work sheet

The given figure shows the map of Austria in a two-dimensional coordinate system. In this figure, some cities of Austria and its neighbouring country Germany can be seen, such as the cities of Vienna, Graz, Linz, Salzburg, Innsbruck, Munich and many more. The coordinates of the cities do not have to be integers.

Task 1:

- a) Indicate which town one is in, when one starts in Salzburg, the move along vector $\vec{u} = (4,5|-2)$ and then along vector $\vec{v} = (5|0)$.

- b) Use vector calculation to determine the distance between Vienna and Graz.

- c) Consider the distance between the cities Munich M and Vienna V . The segment MV is divided by the point T in the ratio 5: 4. Set up an equation to calculate the point T and indicate at which place (town) this point is (approximately) located on the map.

$T =$ _____ Town: _____

Task 2:

- a) Set up a parametric representation of a line between the cities of Salzburg and Vienna. Use a directional vector that has integer coordinates.

- b) Set up an equation (normal vector representation) of the line from task 2a)

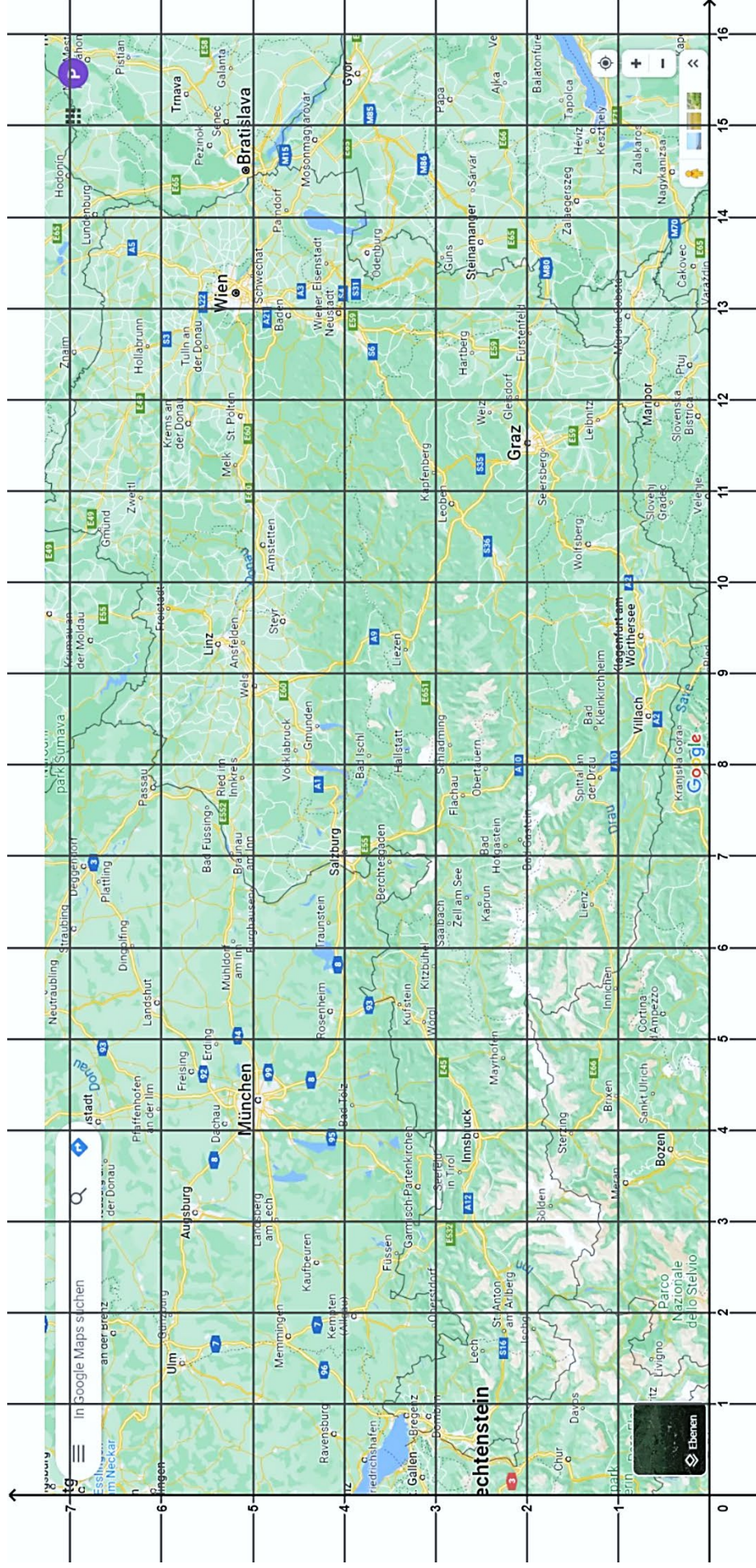
- c) Line g , passing through the cities of Munich and Linz, is given by:

$$g: X = \begin{pmatrix} 4.3 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 25 \\ 2 \end{pmatrix}$$

- 1) Set up a line h which is parallel to g and passes through the city of Salzburg.

- 2) Set up a line i which is normal to g and passes through the city of Salzburg.

Map of Austria



1 Unit in the coordinate system corresponds to approx. 52 km in reality

Calculations with vectors in Austria – Work sheet with solutions

The given figure shows the map of Austria in a two-dimensional coordinate system. In this figure, some cities of Austria and its neighbouring country Germany can be seen, such as the cities of Vienna, Graz, Linz, Salzburg, Innsbruck, Munich and many more. The coordinates of the cities do not have to be integers.

Task 1:

- d) Indicate which town one is in, when one starts in Salzburg, the move along vector $\vec{u} = (4,5|-2)$ and then along vector $\vec{v} = (5|0)$.

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- e) Use vector calculation to determine the distance between Vienna and Graz.

Coordinates of Vienna: $V = (13.2|5.2)$ and Graz $G = (11.5|2)$

$$\overrightarrow{VG} = G - V = \begin{pmatrix} 11.5 \\ 2 \end{pmatrix} - \begin{pmatrix} 13.2 \\ 5.2 \end{pmatrix} = \begin{pmatrix} -1.7 \\ -3.2 \end{pmatrix} \text{ and } \left| \begin{pmatrix} -1.7 \\ -3.2 \end{pmatrix} \right| = \sqrt{(-1.7)^2 + (-3.2)^2} = \sqrt{13.13} \approx 3.62$$

Distance in reality: $3.62 \cdot 52 \text{ km} = 188.24 \text{ km}$

- f) Consider the distance between the cities Munich M and Vienna V . The segment MV is divided by the point T in the ratio 5: 4. Set up an equation to calculate the point T and indicate at which place (town) this point is (approximately) located on the map.

$$T = M + \frac{5}{9} \cdot \overrightarrow{MV} \quad \text{Town: Ansfelden}$$

Task 2:

- a) Set up a parametric representation of a line between the cities of Salzburg and Vienna. Use a directional vector that has integer coordinates.

Coordinates of Vienna $V = (13.2|5.2)$ and Salzburg $S = (7|4)$

$$\overrightarrow{SV} = V - S = \begin{pmatrix} 13.2 \\ 5.2 \end{pmatrix} - \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 6.2 \\ 1.2 \end{pmatrix} \parallel \begin{pmatrix} 62 \\ 12 \end{pmatrix} \parallel \begin{pmatrix} 31 \\ 6 \end{pmatrix}$$

$$u: X = \begin{pmatrix} 7 \\ 4 \end{pmatrix} + r \cdot \begin{pmatrix} 31 \\ 6 \end{pmatrix} \quad \dots \text{ Parametric representation of line } u$$

- b) Set up an equation (normal vector representation) of the line from task 2a)

Directional vector of line u is $\begin{pmatrix} 31 \\ 6 \end{pmatrix}$. A normal vector of line u is hence $\begin{pmatrix} 6 \\ -31 \end{pmatrix}$. Line equation: $\vec{n} \cdot X = \vec{n} \cdot S$

$$\begin{pmatrix} 6 \\ -31 \end{pmatrix} \cdot X = \begin{pmatrix} 6 \\ -31 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 4 \end{pmatrix} \rightarrow 6x - 31y = 6 \cdot 7 + (-31) \cdot 4 \rightarrow 6x - 31y = -82$$

- c) Line g , passing through the cities of Munich and Linz, is given by:

$$g: X = \begin{pmatrix} 4.3 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 25 \\ 2 \end{pmatrix}$$

- 1) Set up a line h which is parallel to g and passes through the city of Salzburg.

$$h: X = \begin{pmatrix} 7 \\ 4 \end{pmatrix} + r \cdot \begin{pmatrix} 25 \\ 2 \end{pmatrix}$$

- 2) Set up a line i which is normal to g and passes through the city of Salzburg.

$$i: X = \begin{pmatrix} 7 \\ 4 \end{pmatrix} + r \cdot \begin{pmatrix} 2 \\ -25 \end{pmatrix}$$

2. Interview Analysis

The teacher teaching this unit has more than 30 years teaching experience. Her second subject is Geography. She did not receive any particular training about teaching in multicultural classrooms.

The teacher considered the timeframe suggested (50 minutes) as sufficient for the performance of all activities. Due to her extensive experience as a Geography teacher, she included interesting facts in the lesson about the map and the cities that were mentioned in the various activities. She also commented on the fact that the chosen grid length of the map (52 km) is of course not a realistic one in real-life maps, but that this length was chosen so that the places that are used in the activities are (at least approximately) at grid points, and so their coordinates are easy to read off by the students.

The teacher also commented positively on the fact that this unit introduced to students places in Austria that are less well-known. Also the fact that vector calculation is used in a real-life setting was positively mentioned.

The possible extension (working with maps of migrant students' background countries, and having them create their own tasks) was mentioned as a good idea, it would however require at least one more lesson.

3. Improvement Suggestions

The teacher recommended two improvements to the unit:

First, she would add to task 2c) the sub-task that students (if needed with the help of the teacher) should interpret the lines that are to be calculated there. What is their meaning in the given context?

Second, she would suggest to give a concrete method for “comparing the results” as the final activity of the lesson. She particularly suggested to write the correct answer onto the blackboard (and not only use verbal teacher-student-communication as suggested) so that all students have the correct answers available.