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Case Study 04

Basic data

Title of Research Teaching Unit	**Triangles and Quadrangles**
Unit created by	**BORG Deutsch-Wagram (AT)**
Based on Concept	**Basic Euclidean Geometry**
Concept created by	**Fondazione A.R.E.A. (IT)**
Date of teaching	**18.03.2024**
Number of students in class	**21**
Number of migrant students	**15**

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1. Unit Description of Research Teaching Unit

Title	**Triangles and Quadrangles**
Age of students/grade	13 years
Length of the unit	2 x 50 minutes
Mathematical topic	Basic properties of triangles and quadrangles
Prerequisite knowledge	• Basic geometric figures
Materials needed	• Introductory slide (for lesson 1) • Cards for team forming (for lesson 1) • Assignment sheets “Quadrangles and Triangles” (for lesson 1) • Presentations and/or posters created by students (for lesson 2)
Objectives	The student • can name the basic properties of triangles and quadrangles • can name the basic differences between triangles and quadrangles • can name the properties of specific triangles (rectangular, isosceles, equilateral) and specific quadrangles (square, rectangle, parallelogram, rhombus, deltoid, trapezoid) • can identify triangles and quadrangles in pictures of everyday life situations • can cooperate with other students to solve problems
Notes on work with migrant students	• Students (migrant and non-migrant) can bring pictures from stations of their lives to find triangular and quadrangular forms in them

Lesson plan:

Lesson 1 (50 minutes) – Introduction of quadrangles and triangles

Time	Method	Content and comments
ca. 5 min	Exposition and Teacher-student-communication	Students are shown various pictures from everyday life (see "introductory slide", and/or pictures students have brought with them). They are asked to guess what the topic of the coming lessons would be.
ca. 10 min	Teacher-student-communication	Brainstorming: "Which quadrangles/triangles do you know". Students can come to the blackboard, write the name of a quadrangle/triangle they know, and make a sketch of it
ca. 25 min (6x3 min work time, plus time for changing stations)	Station learning in groups	Preparation: - Students form 6 teams (students draw cards) - 6 tables are prepared for station learning - Assignment sheets are placed on the 6 tables Students work in teams for 3 minutes per station and note down their answers. The teacher notifies the teams when it is time to switch, then all team switch to the next station.
ca. 5 min	Assignment – homework	Each team draws the name of a specific quadrangle / triangle from a bag/box/… The following names are available: - Rectangle and square - Parallelogram - Rhombus - Deltoid - Trapezoid - Rectangular triangle - Isosceles and equilateral triangle Students are given the following assignment as homework: Research all the important details about the square or triangle you have drawn, such as - What makes it special? Are there any special features regarding side lengths, angles, parallelism, symmetry ... - Which important equations apply to this quadrilateral or triangle? - Explain the calculation of area and circumference? - Is it related to other quadrilaterals/triangles? What do they have in common? Students should give a short 5-minute presentation on this in the coming lesson as a team. Preferably with a poster or PowerPoint-presentation (students can choose which). In addition, they are to prepare a handout with all the important information on it, which will be distributed to all other students in the next lesson
ca. 5 min	Teacher-student-communication	Students can ask questions about the learning stations and/or about the homework assignment

Lesson 2 (50 minutes) – Specific quadrangles and triangles

Time	Method	Content and comments
ca. 5 min	Exposition	Teacher schedules the order of the presentation of the various teams.
ca. 35 min	Team presentations	The students present the results of their assignment (see above) in the following order: - Team “Rectangle and square” - Team “Parallelogram” - Team “Rhombus” - Team “Deltoid” - Team “Trapezoid” - Team “Rectangular triangle” - Team “Isosceles and equilateral triangle”
ca. 10 min	Teacher-student-communication	All important properties and equations are repeated and summarized

Introductory slide

Cards for team forming

Rectangle and square	Parallelogram and rhombus
Deltoid	Trapezoid
Rectangular triangle	Isosceles and equilateral triangle

Assignments for station learning
(summary for teachers)

Description of stations		
Station	Preparation and materials	Description and tasks
Station 1: Properties of quadrangles	Cards with different quadrangles on them are laid out.	The students should work together to describe and discuss the properties of the quadrangles shown.
Station 2: Classifying quadrangles	Cards with different quadrangles on them are laid out.	The students should classify the quadrangles shown, e.g.: Is a square a rectangle? Why, or why not?
Station 3: Constructing quadrangles	Drawing paper, ruler and compasses are provided at the table.	The students should draw different quadrangles and measure the properties such as lengths and angles.
Station 4: Puzzles	- See work sheet	The students are given mathematical puzzles about quadrangles and triangles. The students should solve the puzzle.
Station 5: Constructing triangles	Drawing paper, ruler and compasses are provided at the table.	The students should draw different triangles and measure the properties such as lengths and angles.
Station 6: Similarity of triangles	Cards with different triangles on them are laid out.	The students examine the different pictures of triangles and identify pairs of triangles. They should establish criteria for recognising whether two triangles are similar.

Assignments for station learning
(for students; to be distributed on the 6 tables, one assignment per table)

STATION 1: Properties of quadrangles

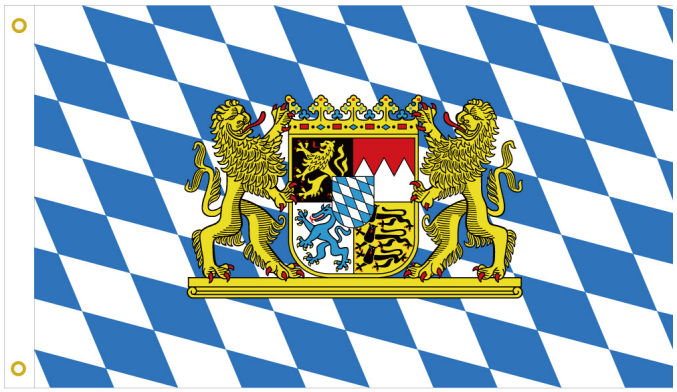
Task:
Describe the properties of the quadrangles shown and discuss them together as a team. Mention the number of sides, angles, side lengths and possible symmetries as well as the parallelism of sides.

STATION 2: Classifying quadrangles

Task:
Try to classify the quadrangles shown and answer the following questions:

- Is every square also a rectangle?
- Is a rhombus also a parallelogram?
- Is every quadrangle a deltoid?
- Is a square also a rhombus?
- Is a parallelogram also a rectangle?

Can you find any other connections or classifications?



STATION 3: Constructing quadrangles

Task:
Try to draw different quadrangles using the enclosed instructions.

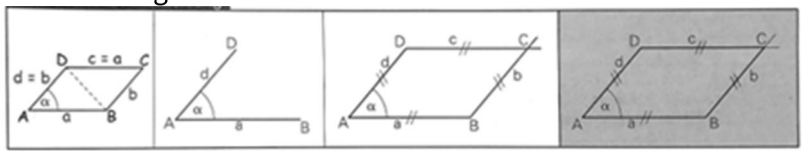
E.g.: Parallelogram: a = 5 cm / b = 3 cm / α = 45°

Construction of a parallelogram / a rhombus

(see *Expedition Mathematik 2, p 239-240*)

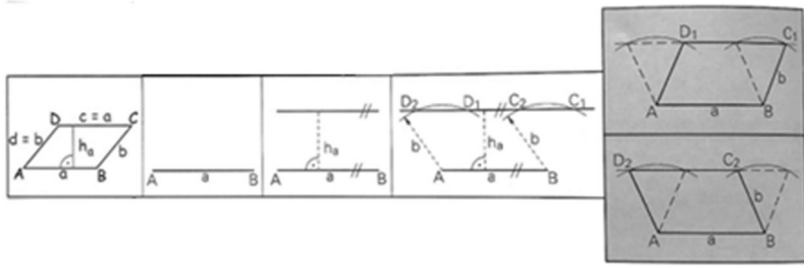
Given: a, b, α

1. Draw side a
2. Draw angle α at vertex A
3. Measure (or transfer) side b (one gets vertex D and side d)
4. Construct side c by parallel translation of a
5. Construct side b by parallel translation of d
6. One gets vertex C



Given: a, h_a, α

1. Draw side a
2. Draw height h_a (normal to a) in the given length
3. Draw a line parallel to a through the end point of the height by parallel translation of a
4. Measure side b with the compass
5. Use the compass first in vertex A, then in vertex B, to transfer the length of b and thereby construct the vertices C and D (one gets 2 solutions)

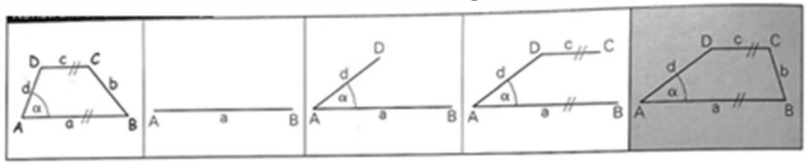


Construction of a trapezoid

(see *Expedition Mathematik 2, p 246*)

Given: a, c, d, α

1. Draw side a
2. Draw angle α at vertex A
3. Measure (or transfer) side d (one gets vertex D and side d)
4. Construct side c by parallel translation of a and using the given length of c (one gets vertex C)
5. Connect vertices C and B (one gets side b)

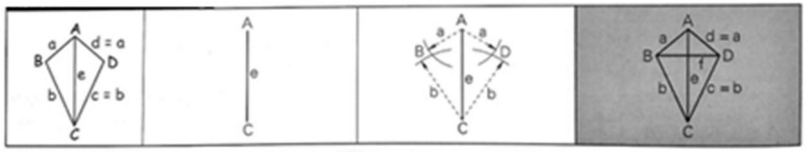


Construction of a deltoid

(see *Expedition Mathematik 2, p 246*)

Given: a, b, e

1. Draw diagonal e
2. Measure length a with the compass, put the compass needle into vertex A and transfer length a on both sides
3. Measure length b with the compass, put the compass needle into vertex C and transfer length b on both sides
4. Intersect compass lines to get vertices B and D
5. Connect vertices to form the sides of the deltoid



STATION 4: Puzzles

Task:
Try to solve the following puzzles together as a team. Which figures are referred to in the following statements? Can there sometimes be more than one answer?

FIGURE 1: A rectangle with sides of equal length.

FIGURE 2: A specific parallelogram with equally long sides.

FIGURE 3: Opposite sides are parallel!

FIGURE 4: It has three vertices and two equally long sides.

FIGURE 5: It has three sides and a right angle.

FIGURE 6: The diagonals are normal to each other.

FIGURE 7: At least two sides are parallel.

FIGURE 8: In this parallelogram adjacent sides are normal to each other.

FIGURE 9: All three sides are equally long.

FIGURE 10: At least one diagonal is cut in half by the other diagonal.

FIGURE 11: Two pairs of parallel sides.

STATION 5: Constructing triangles

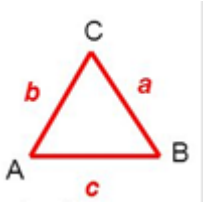
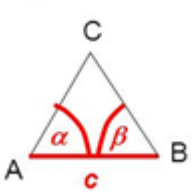
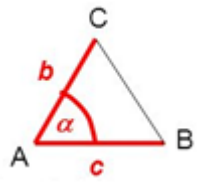
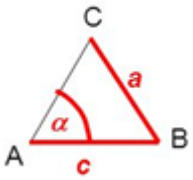
Task:
Try to draw different triangles with the help of the enclosed instructions.
You may use any of the tools provided on the table.

Triangle 1: a = 6 cm / b = 7 cm / 8 cm

Triangle 2: c = 35 mm / b = 43 mm / $\alpha = 55°$

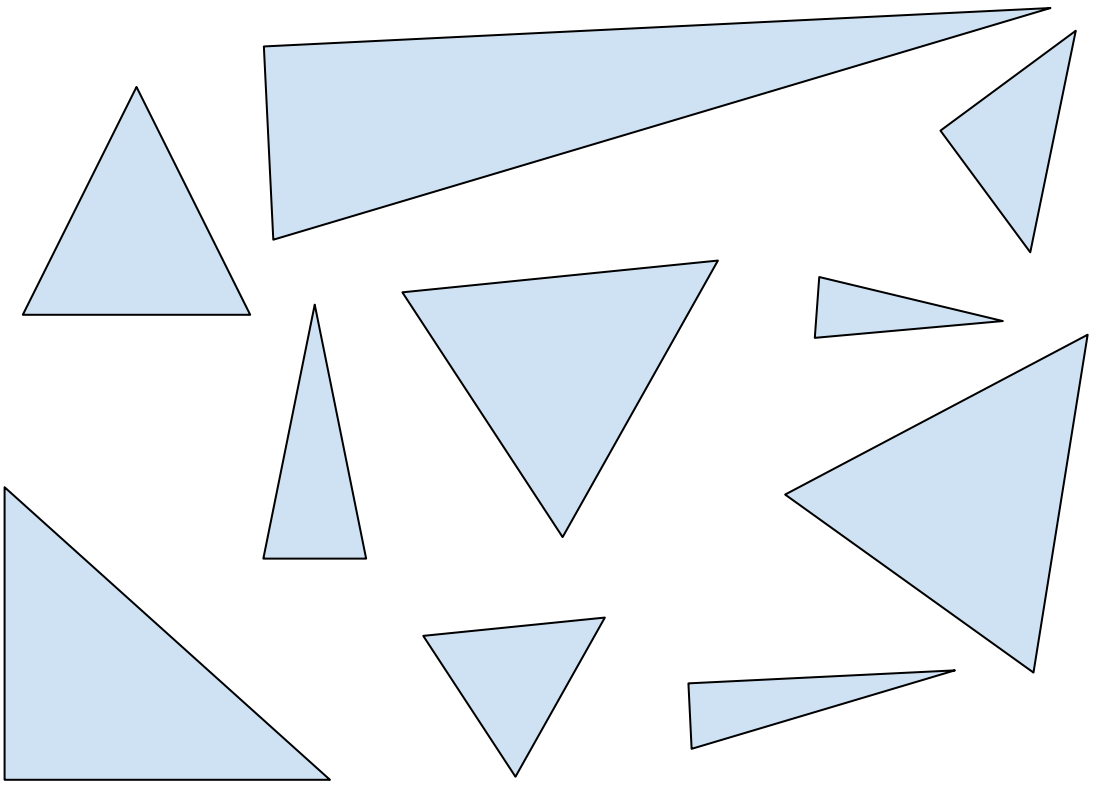
Triangle 3: c = 42 mm / $\alpha = 40°$ / $\beta = 70°$

Triangle 4: c = 30 mm / b = 36 mm / $\beta = 45°$

side-side-side	angle-side-angle	side-angle-side	Side-side-angle
Example: c = 5 cm a = 6 cm b = 6 cm  Construction: - Draw side c - Draw an arc of radius b with centre A - Draw an arc of radius a with centre B - Intersect the two arcs to get vertex C	Example: c = 5 cm α = 60° β = 70°  Construction: - Draw side c - Draw angle α in vertex A and mark its end point - Draw angle β in vertex B and mark its end point - Connect vertices A and B with the corresponding marks by lines - Intersect the lines to get vertex C	Example: c = 5 cm α = 60° b = 6.5 cm  Construction: - Draw side c - Draw angle α in vertex A and mark its end point - Connect vertex A with the mark by a line - Draw an arc of radius b with centre A - Intersect the arc with the line to get vertex C	Example: c = 5 cm α = 60° a = 7 cm  Construction: - Draw side c - Draw angle α in vertex A and mark its end point - Connect vertex A with the mark by a line - Draw an arc of radius a with centre B - Intersect the arc with the line to get vertex C

STATION 6: Similar triangles

Task:
Try to identify pairs of similar triangles shown here. Are there any? As a team, establish criteria that will help you to recognise similar triangles!



2. Interview Analysis

The teacher teaching this unit has 11 years teaching experience. Her second subject is History. She participated in an awareness workshop about teaching in multicultural classrooms in 2022.

The teacher liked the idea of using everyday life objects to introduce the topic. She mentioned she tried similar introductions for other topics, but that it is sometimes hard to come up with appropriate real-life content. She also commented positively on the large variety of real-life objects. While teaching this unit, she added the task to “look around in the classroom for further quadrangles/triangles”.

The teacher commented very positively about the idea of station learning. She knew about the concept (from a course about Montessori materials), but has never tried it in her own classrooms before. She said she was not sure at the beginning whether station learning would work with 13-year-olds (in the Montessori course, they had only covered materials for elementary school, i.e. 6-10-year-olds), but was positively surprised how enthusiastically the students embraced this method. She said the students worked intensely at the stations, and it was “hard to make them stop and go to the next station”.

The teacher said she also was not sure how well the assigned homework would work. Particularly she was not sure whether the instructions in the materials were sufficient for the students to carry out the assigned task. It turned out that the hardest part of the homework (to be done in teams) was the actual organisation of the teamwork outside of the classroom. Students are not used to do homework in teams, and only one team was able to properly organise this. The other teams distributed the work before they left school that day, and then worked independently. They then gave single-student-presentations about their part of the work, and did not make a team presentation as such. The teacher recommended to change this assignment.

3. Improvement Suggestions

The teacher recommended two improvements to the unit:

First, she would suggest to give a task for students to “bring any small fabric, paper, object …, containing geometric patterns with them to the classroom” for the first lesson. This can then be used to find further quadrangles and triangles.

The second suggestion was about the homework assignment at the end of lesson 1. The teacher recommended that this should either be changed to single-student work (each student could e.g. cover only certain aspects of the geometric figure he/she drew), or it should be changed into a team activity that takes place in the classroom at lesson 2 (and then have another lesson where the teams can present their results).