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Case Study 05

Basic data

Title of Research Teaching Unit	**Patterns and geometric transformations in the plane**
Unit created by	**BORG Deutsch-Wagram (AT)**
Based on Concept	**Isometries of the plane**
Concept created by	**Fondazione A.R.E.A. (IT)**
Date of teaching	**21.03.2024**
Number of students in class	**20**
Number of migrant students	**14**

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1. Unit Description of Research Teaching Unit

Title	**Patterns and geometric transformations in the plane**
Age of students/grade	10/11 years
Length of the unit	2 x 50 minutes
Mathematical topic	Axial and point symmetric transformations
Prerequisite knowledge	• Basic geometric concepts • Geometric figures
Materials needed	• Info sheet 1 (for lesson 1) • Task sheet 1 (for lesson 1) • Info sheet 2 (for lesson 2) • Task sheet 2 (for lesson 2)
Objectives	The student • knows the concepts of axial and point symmetry • knows the concept of perpendicular bisection • can recognize axial and point symmetry in given figures and objects • can complete partially given figures by applying axial or point symmetry • can construct a perpendicular bisector of a given line segment
Notes on work with migrant students	• In task 4, migrant students are asked to draw the flag of their country of birth or the country of their family origin. Students considering themselves non-migrant students can e.g. draw the flag of a country where family members live or used to live

Lesson plan:

Lesson 1 (50 minutes)

Time	Method	Content and comments
ca. 10 min	Exposition and Teacher-student-communication	The teacher introduces the concepts of axial and point symmetry (info sheet 1). Students can ask questions, and either copy the definitions into their notebook, or they receive info sheet 1 as a handout.
ca. 20 min	Partner work	Students work in pairs on tasks 1 and 2.
ca. 10 min	Small group work	Students work in small groups on task 3.
ca. 10 min	Small group work, then presentation for whole class	Students work in small groups on task 4 and present their results to the whole class.

Homework: Go to your kitchen and look for utensils or appliances with one or more symmetry properties. Think about whether it is an axial or point symmetry. Sketch the objects onto a piece of paper and bring it with you to the next lesson.

Lesson 2 (50 minutes)

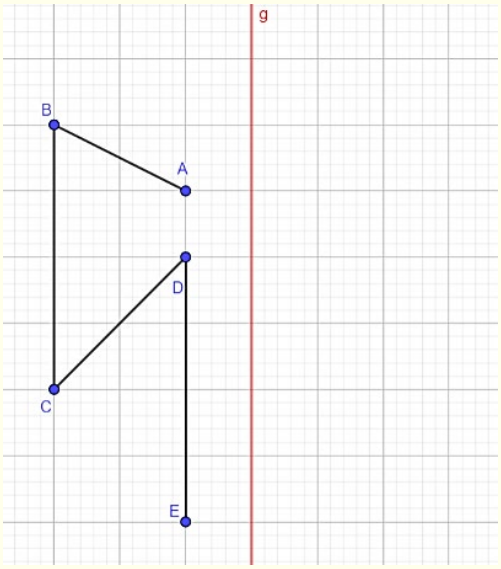
Time	Method	Content and comments
ca. 10 min	Small group work	Students work in small groups on task 5, using the results of the homework. This provides a connection to real-life situations.
ca. 15 min	Small group work, then presentation for whole class	Students work in small groups on task 6 and present their results to the whole class.
ca. 10 min	Exposition and Teacher-student-communication	The teacher introduces the concept of perpendicular bisection (info sheet 2). Students can ask questions, and either copy the information into their notebook, or they receive info sheet 2 as a handout.
ca. 15 min	Individual work, teacher-student-communication	Students work on task 7 individually. Then one student can present the results of a) on the blackboard, and another one the results of b).

Info sheet 1

- **Axial symmetry:**
  Figures that can be made to coincide with themselves by reflecting them on a straight line are called axially symmetrical. The straight line is the axis of symmetry.

  Example:

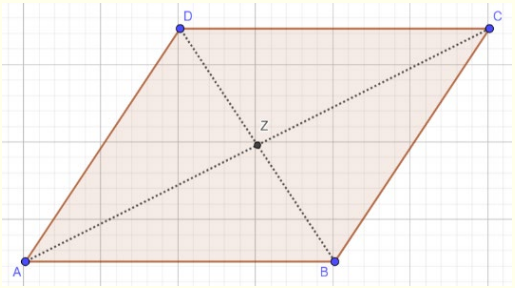
  Using a triangle ruler, construct a mirror image of the given figure, using line g as an axis of symmetry.
  Name the vertices accordingly, i.e. the mirror vertex of is called A', the mirror vertex of B is B' etc.



- **Point symmetry:**
  If a figure is not symmetrical with respect to an axis but with respect to a point, it is referred to as point symmetry. Figures that can be made to coincide with themselves by reflecting them at a point are called point-symmetrical.
  A point-symmetrical figure has a centre of symmetry around which the figure can be rotated by 180°. The rotated figure is congruent with the figure in the starting position.

  Example: Parallelogram

  Z is the centre of symmetry, vertices A and C, and vertices B and D are symmetric with respect to Z; also, all points on one side of the parallelogram are symmetric to a point on the opposite side of the parallelogram.



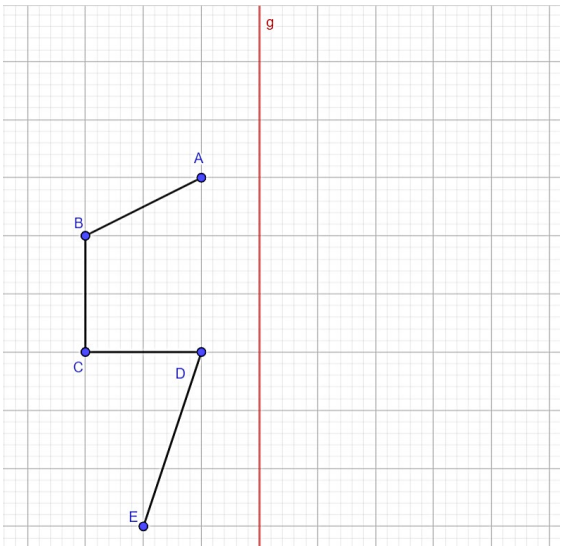
  Special case: Each circle is point symmetric and rotational symmetric with respect to its centre point. Each line through the centre is a symmetry axis, i.e. a circle has infinitely many symmetry axes.

Task sheet 1

Task 1 (partner work):

- Complete each picture to form an axially symmetrical figure. Use the red line as symmetry axis and label the new points A‘, B’, etc.!
- Cut out the fourth figure after mirroring it and check whether it is actually symmetrical by folding it. Think about the role played by the fold line!

a)



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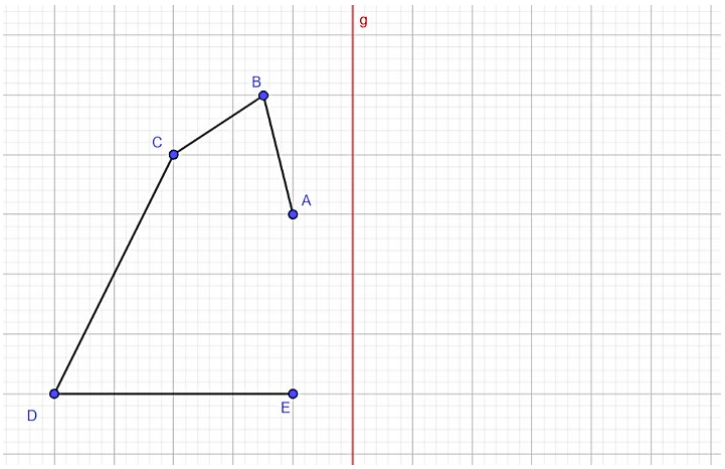

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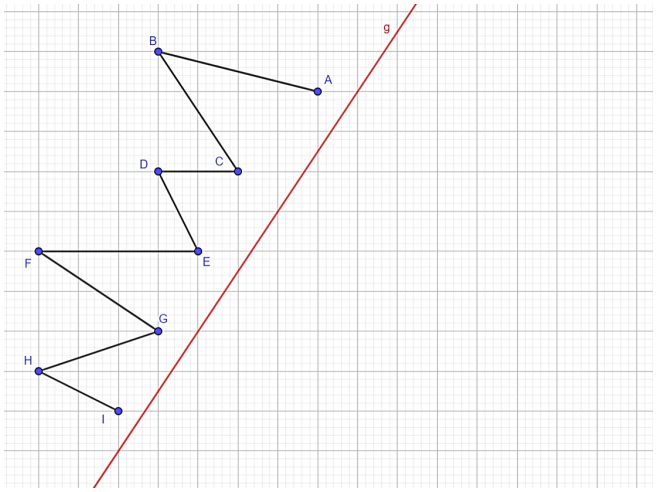

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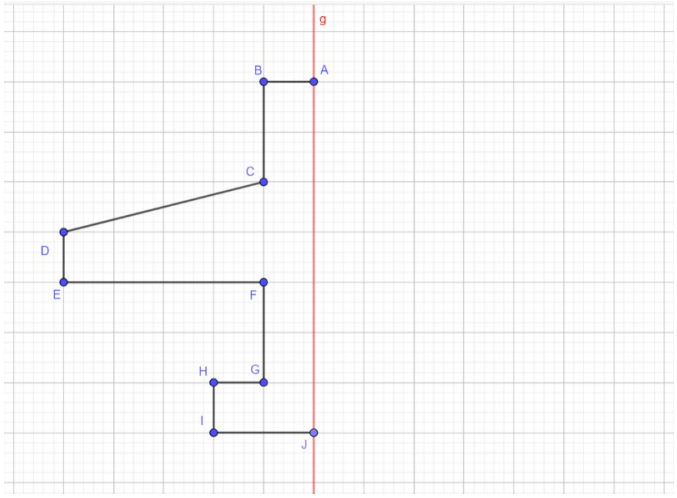
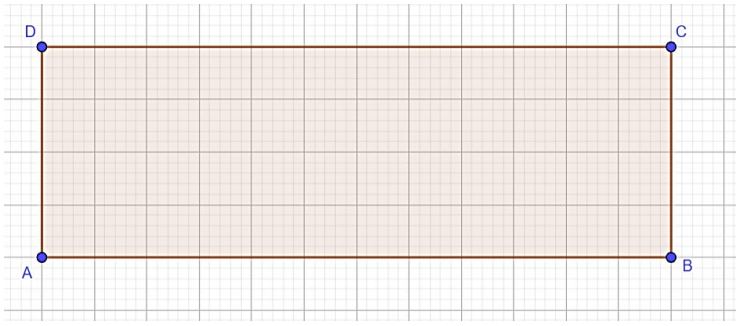


Task 2 (partner work):

Consider whether the figures are point-symmetrical and, if so, draw the centre of symmetry (S).

a)



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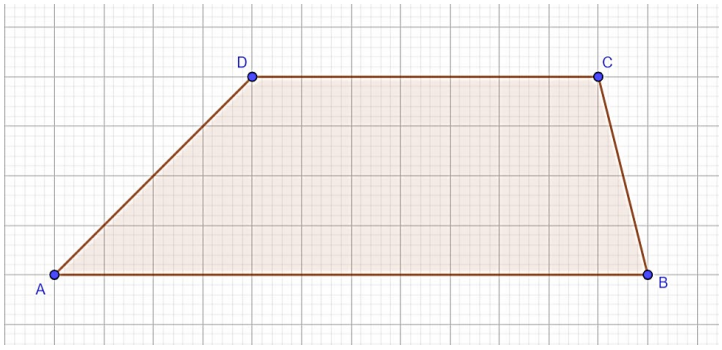

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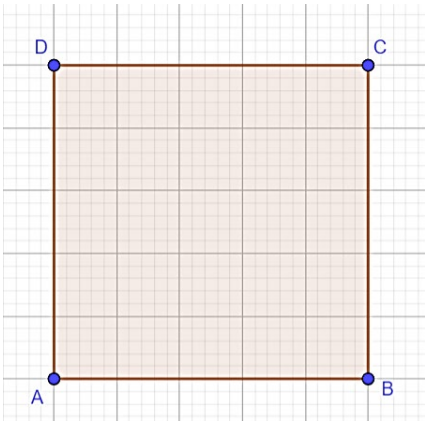

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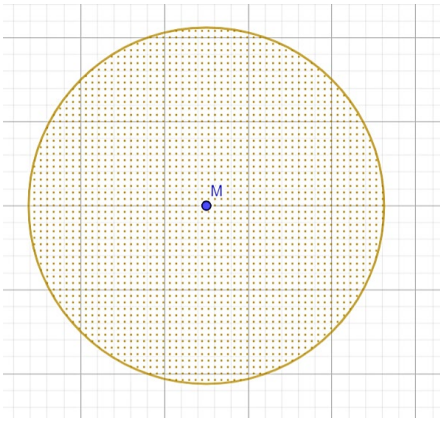


Task 3 (in small group work):

a) Draw three geometric figures with only one axis of symmetry!

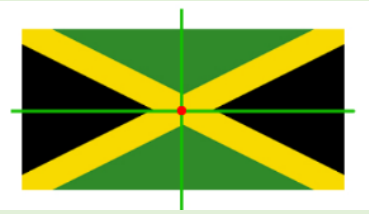
b) Draw three geometric figures with more than one axes of symmetry!

Task 4 (first in small group work, then presentation of results for the whole class):

Sketch the flag of your country of birth on a sheet of paper and consider whether it is axisymmetric or point-symmetric. If it is axially symmetrical, draw all the axes of symmetry on the flag. Then collect as many different flags as possible from the whole class, draw them in colour on an A4 sheet of paper and draw in the axes of symmetry. Ask the teacher to visibly place all the collected flags in the classroom (e.g. on a pinboard).

Note: If there is only one flag for the country of birth in a group, students can sketch the flag of a country where some family members live, or a country they have visited

Example:
The Jamaican flag is axially symmetric with two axes of symmetry.
It is also point-symmetrical.



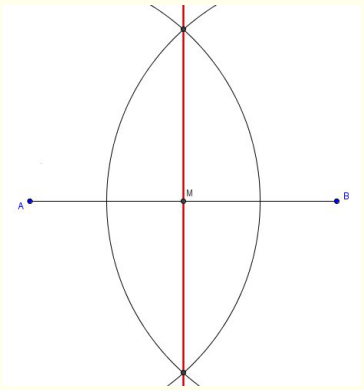
Info sheet 2

Perpendicular bisector

The perpendicular bisector bisects ("cuts in half") the line segment AB and is normal to the line segment AB. All points on the perpendicular bisector are equidistant from points A and B.

Construction:
Points A and B are connected with a line segment.
An arc is drawn around points A and B using a compass.
The radius of the arc must be the same in both cases,
And it must be greater than half the length of the line segment.
The perpendicular bisector then runs through the intersection points of the two arcs.

The perpendicular bisector is also the axis of symmetry of AB.



Task sheet 2

Task 5: „Let's bake biscuits!" (small group work)

a) Which of these biscuit cutters have axes of symmetry and which do not?
If there is an axis of symmetry, draw it in colour!
Are there biscuit cutters that are point-symmetrical? If so, which ones?

b) Look at the kitchen utensils (or their sketches) you brought with you (which was the homework). Discuss in your group whether they are axially symmetric or point-symmetric!

c) Students can bring their favourite biscuit recipe with them and distribute them to the other group members. Motivated students can bring self-baked biscuits with symmetric shapes to the next lesson and distribute them.

Task 6 (first in small group work, then in the whole class):

a) There are words that are axially symmetric, e.g. TOOT, BOB or EVE.

Each student in the group comes up with two axially symmetric words in their first language.
The internet can be used as a resource.
The words shall then be shown and their meaning explained to the other group members, and their translation into the language of instruction shall be introduced.

Example:

EBBE (axially symmetric word in German):
Explanation: Regularly recurring fall in sea level, which becomes visible as the sea recedes.
Translation: Low tide

b) There are words that being read from back to front also form meaningful words. Such words are called palindromes (note: sometimes, "palindrome" is used in a strict sense meaning only axially symmetric words, i.e. words that read the same from back to front as from front to back; here we use "palindrome" in a broader sense), e.g. TEN – NET, EVIL – LIVE

Each student in the group comes up with two palindromes in their first language.
Again, The internet can be used as a resource.
The words shall then be shown and their meaning explained to the other group members, and their translation into the language of instruction shall be introduced.

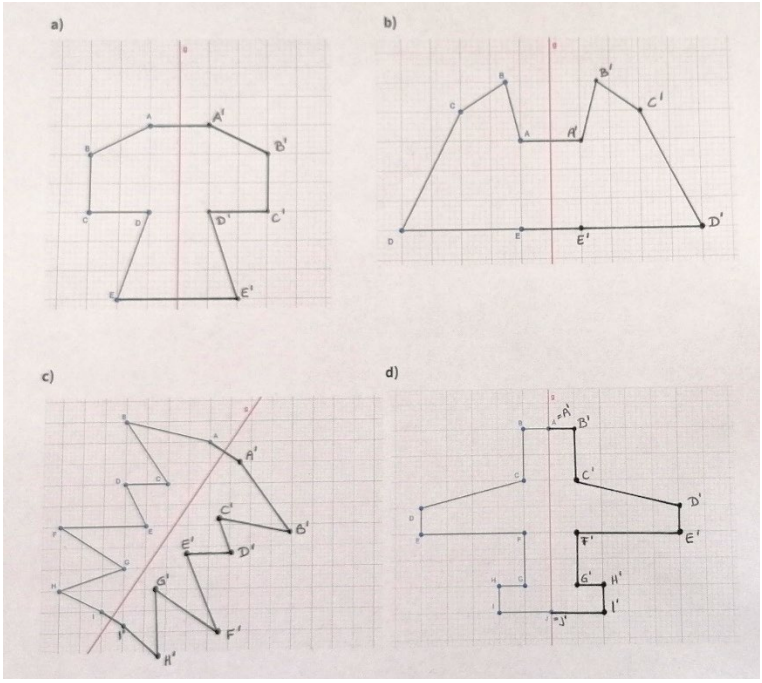
Task 7:

a) Construct the perpendicular bisector of the line segment AB, where $A = (0|7)$ and $B = (6|4)$.

b) Given are the points $C = (2|5)$ and $D = (12|7)$. Enter the coordinates of the centre point M of the line segment CD. Then divide the line segment CD into four parts of equal length using perpendicular bisection.

Solutions

Task 1:



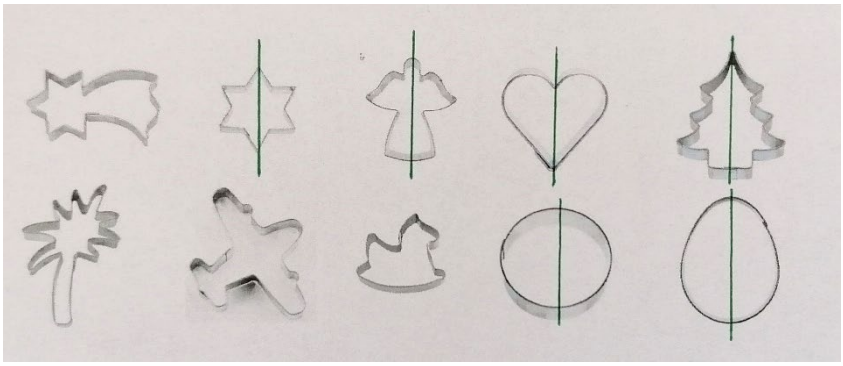
Task 2:

Figures a) c) and d) are point-symmetrical.

Task 5:

a) The following biscuit cutters have at least one axis of symmetry: Star, angel, heart, tree, circle, egg
The following biscuit cutters are (also) point-symmetrical: Star, circle

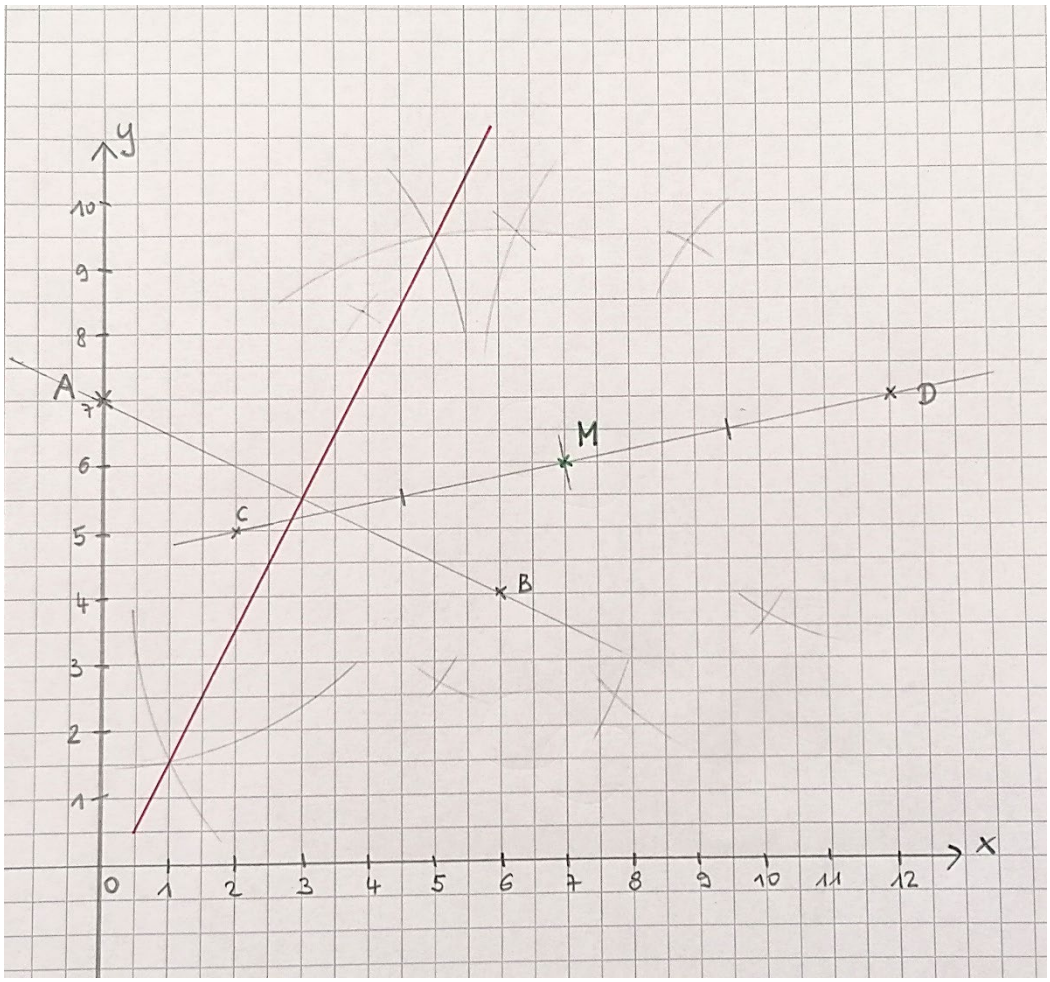
Axes of symmetry:



Task 7:

The centre point has the coordinates $M = (7|6)$.

Construction:



2. Interview Analysis

The teacher teaching this unit has 11 years teaching experience. Her second subject is History. She participated in an awareness workshop about teaching in multicultural classrooms in 2022.

The teacher commented that the introduction of axial and point symmetry is very similar to what she did in past lessons when she had to introduce those topics. She said that axial symmetry is a well-known concept to students, even if they do not know the formal definition and properties. Point symmetry is harder to understand, since it is not something students usually experience in their daily lives. She recommended to use GeoGebra as an addition to drawing on the board.

The teacher liked task 3, because “it got the students to think, it is a reverse task in a way”. Most tasks that she has used before were like tasks 1 and 2 (complete a figure, find symmetry in a given figure), while task 3 requested that students find their own figures.

The teacher commented particularly positively on task 4. “Working with the flags was a good idea, and using their own flags is brilliant”. She said while students sometimes do not like to talk about their countries of origin, the topic of geometry of the flags allowed them to bring context into the classroom and contribute things that other students would not know, without the need of having to discuss the political or economic issues that often come up when their countries of origin are mentioned.

The teacher also liked task 5, and commented on how enthusiastically the students embraced the idea of bringing cooking utensils into the classroom. Almost all students brought something, one boy brought a large collection of biscuit cutters, and one girl brought various cake decorations. Students worked on this task longer than the assigned time, and the teacher decided to give them this extra time.

The teacher considered task 6 to be hard. During the lesson, it turned out it was difficult for students to find “symmetrical words” in any language. While they liked the idea, students grew a bit frustrated about their inability to come up with words. She suggested this might need more preparation time, perhaps as homework.

Finally, the teacher did not use task 7, as she ran out of time. She commented that she would not have used it anyway, because it was a “very traditional and abstract” task compared to the rest of the activities in lesson 2.

3. Improvement Suggestions

The teacher recommended several improvements to the unit:

First, her suggestion was to use GeoGebra for the constructions, in addition to the board and students' notebooks. She considered it important that students can make these constructions by hand, but also show them how they can use technology to check on their work.

The second suggestion was about task 6 (symmetrical words). She considered giving the students part of this task as homework (find symmetrical words in your first language), and then have them presented and discussed in the lesson. In this way, students can use different resources (not only online, but also asking family members and/or friends). It would also reduce the amount of time used on task 6, which she recommended to be better used for task 5.

Finally, the teacher suggested to either cancelling task 7, or move the entire topic of perpendicular bisection to the next lesson. This was suggested for two reasons. First, there already are many time consuming activities in lesson 2, and it is difficult to find enough time for properly doing task 7. Second, she thought that this kind of task (construction activity) does not fit with the rest of the activities in this lesson.