



Case Study 06

Basic data

Title of Research Teaching Unit	Representation of solids in the plane
Unit created by	BORG Deutsch-Wagram (AT)
Based on Concept	Isometries of the plane
Concept created by	Fondazione A.R.E.A. (IT)
Date of teaching	22.03.2024
Number of students in class	18
Number of migrant students	7

1. Unit Description of Research Teaching Unit

Title	Representation of solids in the plane
Age of students/grade	13 years
Length of the unit	3 x 50 minutes
Mathematical topic	Projections of solids in geometry
Prerequisite knowledge	<ul style="list-style-type: none">• Properties of basic solids (cube, various forms of prisms, cylinder, pyramid, cone, sphere)• Working with equations and variables
Materials needed	<ul style="list-style-type: none">• Info sheet 1 (for lesson 1)• Task sheet 1 (for lesson 1)• Task sheet 2 (for lesson 2)• Info sheet 2 (for lesson 3)• Task sheet 3 (for lesson 3)
Objectives	<p>The student</p> <ul style="list-style-type: none">• knows the equations for the surface area and the volume of cube, cuboid, cylinder, square-based pyramid, cone and sphere• can construct an oblique parallel projection with a given angle and a given factor (foreshortening) of a cube and a cuboid
Notes on work with migrant students	<ul style="list-style-type: none">• For task 4, students (migrant and non-migrant) shall think of a typical object of their (or their family's) country of origin, which shall have a basic geometric shape. Students can briefly explain what this object is, or why they chose this particular object

Lesson plan:**Lesson 1 (50 minutes)**

Time	Method	Content and comments
ca. 30 min	Small group work, then presentation of results to the whole class	Repetition of properties of geometric solids; research about real buildings (internet); connection to mathematics is made (task sheet 1).
ca. 20 min	Exposition and teacher-student-communication	The teacher explains the process of constructing an oblique parallel projection, as well as the equations for the surface area and the volume of a cube and a cuboid, using info sheet 1. Students can ask questions.

Lesson 2 (50 minutes)

Time	Method	Content and comments
ca. 20 min	Whole class work, teacher-student-communication	Solving of task 2 (teacher, together with the whole class).
ca. 30 min	Partner work, whole class work, discussion	Solving of task 3 (in pairs), then checking of solutions in whole class work. Discussion about the “typical objects” chosen in task 3b).

Homework: Each student should think of a typical object of their (or their family's) country of origin that represents a geometric shape. They should also find out the dimensions of this object (e.g. via the Internet or by measuring the object at home; if neither is possible, the students should invent realistic measurements). The students are welcome to take a photo of the object or even bring the selected objects to class.

Lesson 3 (50 minutes)

Time	Method	Content and comments
ca. 20 min	Exposition and teacher-student-communication	The teacher explains the equations for the surface area and the volume of cylinder, square-based pyramid, cone, and sphere, using info sheet 2. Students can ask questions.
ca. 30 min	Partner work, presentation of results to the whole class	The students work on task 4 in partner work (2 students), then present their results to the whole class (if students need more time for the task, then presentation of results may also take place in the next lesson).

Task sheet 1

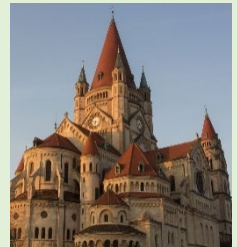
Task 1 (small group work, then presentation of results to the whole class):

Each student in the group thinks of a building/sight from their country of origin that contains geometric solids! The geometric solids should also be named! The use of the Internet is permitted! Each student should also briefly explain “their” building in 2-3 sentences.

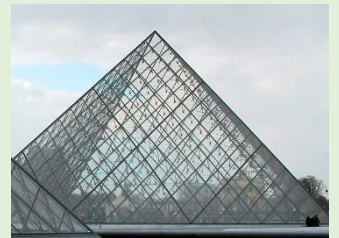
Should all (or most of) students have the same country of origin, they can pick countries where they visited or where family members live.

Examples:

- a)** Church “Saint Francis of Assisi” in Vienna (the peaks of the towers are cones)
It is on Mexikoplatz (Mexico square), directly at Danube River.
The cornerstone has been laid at the 50th anniversary of the enthronisation of emperor Franz Josef. It is a roman catholic parish church.



- b)** Glass pyramid in Paris
It serves as the main entrance of the Louvre Museum in the French capital of Paris.



Info sheet 1

The focus is now on creating an oblique parallel projection, and calculating the surface area and volume of a cube and cuboid:

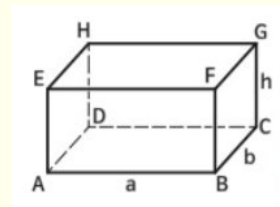
A drawing always shows a solid from only one angle. The solid, i.e. a spatial object, is depicted as a plane figure on the drawing paper. To achieve a spatial effect in the drawing, the so-called *oblique parallel projection* is often used. In this type of representation, individual surfaces and edges of the solid are given different shapes or lengths.

Explanation: The front and rear surfaces are drawn in true size. The side edges running diagonally backwards are shortened. We are working with the distortion angle $\alpha = 45^\circ$ (angle at which the oblique edge is drawn) and the shortening factor, also called *foreshortening*, $v = 0.5$ (i.e. a shortening by half; this is the factor by which the length of the oblique edge is multiplied). Invisible edges are drawn as a dashed or dotted line. This gives a representation that corresponds well to reality.

For a **cuboid** with base area G the following applies:

$$\text{Surface area } A = 2 \cdot a \cdot b + 2 \cdot a \cdot h + 2 \cdot b \cdot h = 2 \cdot G + M$$

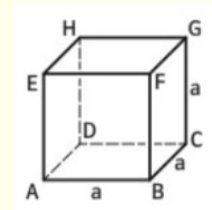
$$\text{Volume: } V = a \cdot b \cdot h = G \cdot h$$



For a **cube** with base area G the following applies:

$$\text{Surface area } A = 6 \cdot a \cdot a$$

$$\text{Volume: } V = a \cdot a \cdot a = G \cdot a$$



Task sheet 2

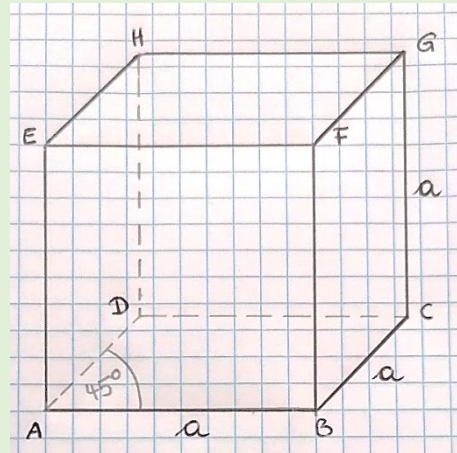
Task 2 (whole class work):

Construct the oblique parallel projection of a cube with side length $a = 5\text{ cm}$.

Calculate the surface area as well as the volume of the cube

$$\text{Surface area } A = 6 \cdot a \cdot a = 6 \cdot 5 \cdot 5 = 150\text{ cm}^2$$

$$\text{Volume: } V = a \cdot a \cdot a = G \cdot a = 5 \cdot 5 \cdot 5 = 125\text{ cm}^3$$



Task 3 (partner work):

- a) Cuboid: Construct the oblique parallel projection of a cuboid with sides $a = 7\text{ cm}$, $b = 4\text{ cm}$, $c = 3\text{ cm}$, using the angle $\alpha = 45^\circ$ and the foreshortening $v = 0.5$.

Calculate the surface area and the volume of this cuboid!

Think of 2 objects from everyday life that could have approximately the shape of this cuboid!

- b) Cube: Construct the oblique parallel projection of a cube with side length $a = 7\text{ cm}$.
Use $\alpha = 45^\circ$ and $v = 0.5$.

Calculate the surface area and the volume of this cube!

Think of 2 objects from everyday life that could have approximately the shape of this cube!

Info sheet 2

To further deepen the topic of geometric solids, the surface areas and volumes of the following solids are now discussed in more detail: Cylinder, square-based pyramid, cone, sphere.

Note: The students are already familiar with the properties of geometric solids. The students are also familiar with the terms $h_a, s, \dots$. Using a matching format, the students should work out for themselves how the corresponding equations look like.

- Cylinder:

Surface area:

Volume:

$$V = \frac{4}{3} \cdot r^3 \cdot \pi$$

$$A = a^2 + 2 \cdot a \cdot h_a$$

- Square-based pyramid:

Surface area:

Volume:

$$V = \frac{1}{3} \cdot r^2 \cdot \pi \cdot h$$

$$A = 2 \cdot r^2 \cdot \pi + 2 \cdot r \cdot \pi \cdot h$$

- Cone:

Surface area:

Volume:

$$V = \frac{1}{3} \cdot a^2 \cdot h$$

$$A = 4 \cdot r^2 \cdot \pi$$

- Sphere:

Surface area:

Volume:

$$V = r^2 \cdot \pi \cdot h$$

$$A = r^2 \cdot \pi + r \cdot \pi \cdot s$$

Task sheet 3

Task 4 (small group work, then presentation of results to the whole class):

Each student should think of a typical object of their (or their family's) country of origin that represents a geometric shape (this was the homework). The following table should be completed in the small group. The results should then be presented on the board to the whole class. Students are welcome to take the selected objects to class and (if they are edible) even have them tasted.

The first line shows an example for a typical object from Austria.

Object	Solid	Dimensions	Number of vertices	Number of edges	Number of faces	Surface area	Volume
 Manner-Schnitten (typical Viennese wafers)	Cuboid	$a = 97$ $b = 84$ $h = 15$ (mm)	8	12	6	$A =$ $= 2 \cdot 97 \cdot 84 +$ $2 \cdot 97 \cdot 15 +$ $2 \cdot 84 \cdot 15$ $= 21726 \text{ mm}^2$	$V = 97 \cdot 84 \cdot 15$ $= 122220 \text{ mm}^3$ $= 122.22 \text{ cm}^3$

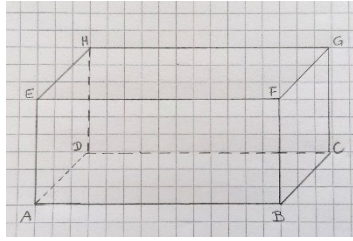
Solutions

Task 3:

a)

$$A = 2 \cdot 7 \cdot 4 + 2 \cdot 7 \cdot 3 + 2 \cdot 4 \cdot 3 = 122 \text{ cm}^2$$

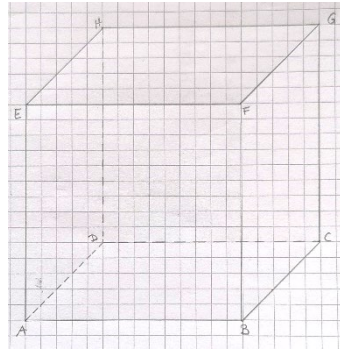
$$V = 7 \cdot 4 \cdot 3 = 84 \text{ cm}^3$$



b)

$$A = 6 \cdot 7^2 = 294 \text{ cm}^2$$

$$V = 7^3 = 343 \text{ cm}^3$$



Info sheet 2:

- Cylinder:

Surface area:

$$A = 2 \cdot r^2 \cdot \pi + 2 \cdot r \cdot \pi \cdot h$$

Volume:

$$V = r^2 \cdot \pi \cdot h$$

- Square-based pyramid:

Surface area:

$$A = a^2 + 2 \cdot a \cdot h_a$$

Volume:

$$V = \frac{1}{3} \cdot a^2 \cdot h$$

- Cone:

Surface area:

$$A = r^2 \cdot \pi + r \cdot \pi \cdot s$$

Volume:

$$V = \frac{1}{3} \cdot r^2 \cdot \pi \cdot h$$

- Sphere

Surface area:

$$A = 4 \cdot r^2 \cdot \pi$$

Volume:

$$V = \frac{4}{3} \cdot r^3 \cdot \pi$$

2. Interview Analysis

The teacher teaching this unit has 12 years teaching experience. His second subject is Physical Education (Sports). He did not receive any particular training about teaching in multicultural classrooms.

The teacher commented very positively on task 1 (solids in the real world). He prepared some pictures of buildings to introduce them in the class in case students should not find suitable photos in the internet, but did not use them for the actual lesson, since students were able to find a good number of pictures to be used for this task. He said she would have preferred continuing working with the objects found in task 1, and eventually come back to them after introducing the oblique parallel projection.

The teacher described lesson 2 (tasks 2 und 3) as fairly standard and very similar to what is suggested in the text book she regularly uses for teaching. He said students were fairly creative in task 3b (finding everyday life objects in cube shape), though they mostly named objects that are actually cuboids.

The teacher was not exactly sure how to practically work with info sheet 2. If it is distributed in hard copy (as he originally intended to do), students would have to cut out the equations, match them, and glue them into their notebook. If it is projected onto a screen as a word file and then worked on in a whole-class setting (which he ended up doing), students will have to write all the equations in the right order into their notebooks, which is error-prone. Another possibility would be to work on the file in whole-class setting to establish the correct matching, then distribute a hard copy containing the correct matching to the students.

The teacher said that the students very much enjoyed task 4. Students came up with a number of different objects, and some actually did bring objects into the classroom. He particularly commented on the positive impact of this activity on some of the migrant students. Especially he commented on one boy whom he described as “usually very timid and reserved”. This student brought an object from his family and proudly presented it first to the group and then to the whole class. The teacher said it would be good to make more connections to students’ backgrounds.

3. Improvement Suggestions

The teacher recommended several improvements to the unit:

First, he would suggest to use the objects (buildings with geometric shapes) of task 1 and work with them in lesson 2, i.e. let the students create oblique parallel projections of these objects instead of general abstract ones as it is currently done.

The teacher also suggested to make it more clear how to work with info sheet 2 in lesson 3. He would either work on it in a whole-class setting and then distribute a correct solution of the matching task to the students, or let the students work directly on the electronic file which could be distributed via Moodle. She considered the use of a paper version for the actual matching exercise as impractical. Regardless of the mode of usage, he suggests that students should have an info sheet with the correct matchings at the end of the lesson.