



Case Study 07

Basic data

Title of Research Teaching Unit	Mathematics in the flower bed
Unit created by	BORG Deutsch-Wagram (AT)
Based on Concept	Green Geometry
Concept created by	Istituto Comprensivo Statale “Malaspina” Massa (IT)
Date of teaching	13.05.2024
Number of students in class	11
Number of migrant students	1

1. Unit Description of Research Teaching Unit

Title	Green Geometry 2 (Mathematics in the flower bed)
Age of students/grade	17-18 years
Length of the unit	1 x 50 minutes (up to 2 x 50 minutes)
Mathematical topic	Applications of differentiation and integration
Prerequisite knowledge	<ul style="list-style-type: none">• Setting up the equation of a polynomial function with given function values• Differentiation of polynomial functions• Integration of polynomial functions• Interpretation of the Integral as the area beneath the graph
Materials needed	<ul style="list-style-type: none">• Task sheet 1
Objectives	The student <ul style="list-style-type: none">• can apply knowledge about properties of polynomial functions in practical, real-life situations• can use polynomial functions to model real-life objects• can apply differentiation and integration in real-life contexts
Notes on work with migrant students	<ul style="list-style-type: none">• Students (particularly migrant students) can bring pictures from flower beds in gardens or parks in their own countries of origin

Lesson plan:**Lesson 1 (50 minutes)**

Time	Method	Content and comments
ca. 5 min	Exposition and teacher-student-communication	The teacher motivates the students for the topic of the lesson and explains its significance in everyday life.
ca. 35 min	Partner work, or work in small groups	The students solve the tasks on the worksheet in teams of two or three. The teacher goes round the class and helps with any problems that arise.
ca. 10 min	Teacher-student-communication	Comparison of results

Should the class be very engaged and interested in the tasks of lesson 1, students can work on an additional task (Task 4) either at the end of lesson 1 (time permitting), or in a second lesson.

The shape of the flower bed is described by a function f , where x is the width of the flower bed in metres and $f(x)$ is the height of the flowerbed in metres.

A gardener lays out a customer's flower bed in such a way that the flower bed starts at the edges of the garden and reaches its maximum height in the centre of the garden. The customer notes that the flowerbed should have a maximum 'height' of three metres. Furthermore, it is known that the function f is only defined for $0 \leq x \leq 10$.

- Room for your sketch:

- | Description in words | Mathematical description |
|---|---|
| The flower bed starts at the edges of the garden.
These edges are at $x = \underline{\hspace{1cm}}$ and $x = \underline{\hspace{1cm}}$. | I: <u> </u>
II: <u> </u> |
| The flower bed has its maximum height in the centre of the garden. The centre of the garden is at $x = \underline{\hspace{1cm}}$. | III: <u> </u> |
| The customer notes that the flower bed should have a maximum ‘height’ of three metres. | IV: <u> </u> |

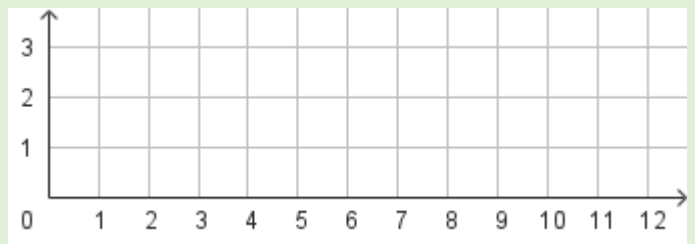
Polynomial function of degree ____: This is generally given by: $f(x) =$ _____

The polynomial function is given by: $f(x) =$ _____

Task 2:

Sketch the polynomial function that describes the flower bed again in the given coordinate system.

Estimate the actual area of the flowerbed using upper sums and lower sums (with 5 equal intervals).

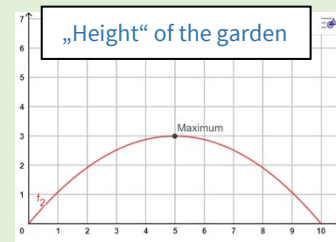
**Task 3:**

Determine the actual total area of the flower bed in the garden.

The shape of the flower bed is described by a function f , where x is the width of the flower bed in metres and $f(x)$ is the height of the flowerbed in metres.

A gardener lays out a customer's flower bed in such a way that the flower bed starts at the edges of the garden and reaches its maximum height in the centre of the garden. The customer notes that the flowerbed should have a maximum 'height' of three metres. Furthermore, it is known that the function f is only defined for $0 \leq x \leq 10$.

- Room for your sketch:



Width of the garden

- | Description in words | Mathematical description |
|---|----------------------------------|
| The flower bed starts at the edges of the garden.
These edges are at $x = 0$ and $x = 10$. | I: $f(0) = 0$
II: $f(10) = 0$ |
| The flower bed has its maximum height in the centre of the garden. The centre of the garden is at $x = 5$. | III: $f'(5) = 0$ |
| The customer notes that the flower bed should have a maximum 'height' of three metres. | IV: $f(5) = 3$ |

Polynomial function of degree 2: This is generally given by: $f(x) = ax^2 + bx + c$

$$f(x) = ax^2 + bx + c \quad \text{und} \quad f'(x) = 2ax + b$$

$$\begin{array}{lll} \text{I: } f(0) = 0 & a \cdot 0^2 + b \cdot 0 + c = 0 & \rightarrow c = 0 \\ \text{II: } f(10) = 0 & a \cdot 10^2 + b \cdot 10 + c = 0 & \rightarrow 100a + 10b = 0 \\ \text{III: } f'(5) = 0 & 2 \cdot a \cdot 5 + b = 0 & \rightarrow 10a + b = 0 \\ \text{IV: } f(5) = 3 & a \cdot 5^2 + b \cdot 5 + c = 3 & \rightarrow 25a + 5b = 3 \end{array}$$

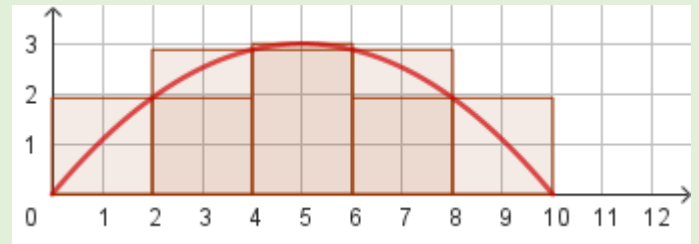
$$\begin{array}{lcl} \text{III:} & 10a + b = 0 & | \cdot (-5) \\ \text{IV:} & 25a + 5b = 3 & \end{array} \quad \begin{array}{lcl} & -25a = 3 & | : (-25) \rightarrow a = -\frac{3}{25} \\ & & \end{array}$$

The polynomial function is given by $f(x) = -\frac{3}{25}x^2 + \frac{6}{5}x$

Task 2:

Sketch the polynomial function that describes the flower bed again in the given coordinate system.

Estimate the actual area of the flowerbed using upper sums and lower sums (with 5 equal intervals).



To estimate the area with the help of upper and lower sums, some function values are required, the values of which are given in the following table of values (rounded).

x	0	2	4	5	6	8	10
$f(x)$	0	1.92	2.88	3	2.88	1.92	0

Lower sum: $L_5 = 2 \cdot [f(0) + f(2) + f(4) + f(8) + f(10)]$
 $L_5 \approx 13.44$

Upper sum: $U_5 = 2 \cdot [f(2) + f(4) + f(5) + f(6) + f(8)]$
 $U_5 \approx 25.2$

One gets: $13.44 \text{ m}^2 < \text{actual area} < 25.2 \text{ m}^2$

Task 3:

Determine the actual total area of the flower bed in the garden.

$$\begin{aligned}
 A &= \int_0^{10} f(x) dx = \int_0^{10} \left(-\frac{3}{25}x^2 + \frac{6}{5}x \right) dx = \left(-\frac{3x^3}{25 \cdot 3} + \frac{6x^2}{5 \cdot 2} \right) \Big|_0^{10} \\
 &= \left(-\frac{3 \cdot 10^3}{25 \cdot 3} + \frac{6 \cdot 10^2}{5 \cdot 2} \right) - \left(-\frac{3 \cdot 0^3}{25 \cdot 3} + \frac{6 \cdot 0^2}{5 \cdot 2} \right) = (-40 + 60) = 20
 \end{aligned}$$

The actual total area of the flower bed is 20 m^2 .



Additional task



★ Task 4:

In order to make the best possible use of the garden area, the customer asks herself what the optimum width x the plot should have to maximise the total area of the flowerbeds in the garden. Determine the optimum width and describe how you did it in a comprehensible way.

2. Interview Analysis

The teacher teaching this unit has 7 years teaching experience. Her second subject is Physics. She did not receive any particular training about teaching in multicultural classrooms.

The teacher mentioned that although the context of the tasks is probably not what an actual gardener would do, she nevertheless liked working with them. The students enjoyed the fact that there was an “unusual context”, as opposed to just function graphs with no context at all or standard optimization tasks.

The teacher also commented on this activity being a good preparation for the secondary schools’ final exam, where tasks with integration and calculation of area beneath graphs are fairly usual. She mentioned that she was originally hesitant with using a non-standard context for this preparation, but she said that the actual mathematical processes used were the same than in her usual preparations, only with a context that was more interesting to students.

For the calculations of the lower and upper sums in task 2, and also for checking the results of tasks 1 and 3, the teacher used GeoGebra. She generally commented on the lack of technology use in the teaching unit, and would suggest to have either some extra tasks where students are to use technology, or to change the given tasks in such a way that the students can practice their use of technology, also with a view towards the final exam.

The teacher also added that it would be nice to eventually use the school yard while working on task 1, to make a “real-life sketch” of the garden.

3. Improvement Suggestions

The teacher recommended several improvements to the unit:

First, she would suggest to add the use of technology, particularly Dynamic Geometry Software, for working on the tasks. Especially the activity of task 2 (calculation of lower and upper sum) she considered as “not very helpful to improve students’ competencies” if done by hand. Students have done such activities before and know how to do them, and in an “applied task” the hand calculations required would only slow them down and decrease their motivation.

Her second suggestion would be to go out of the classroom into the school yard or even a nearby playground, where the students could e.g. use a grassy area or even a sand box to draw a life-sized sketch of their “garden”. She said this would be very motivating for the students, since they can apply higher mathematics in an out-of-doors setting, making them aware that mathematics is literally “all around us”.