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Concepts for teaching units-Mathematical modeling

Title	**Contrasting the nature's economy to overconsumption in the capitalism framework**
Introduction	The proposed activities of this unit adopt the logic of mathematical modeling using digital tools as well as digital material. They are an example of mathematization of a realistic situation concerning the shape of the beehive cells. This topic will be the starting point of a wider framework of dialogues related to the economy of nature with main characteristics the minimum actions, the minimum routes as well as the maximum yields, in contrast to the global capitalist economy with the main characteristics of overproduction, overconsumption as well as the over-pumping of natural resources.
Age of students/grade	Lower-secondary students
Length of the unit	3 x 45 minutes
Materials needed	Computer with the free Geogebra software. Paper, pencil & scissor
Objectives	The teacher through the ideas described in this concept unit will have the opportunity his/her students to: ➢ Familiarize with modelling process. ➢ Discuss topics of energies. ➢ Discuss about the economy of nature (the minimum routes as well as the maximum yields) ➢ Realize the difference of the way nature works in contrast to the global capitalist economy that of the overproduction and overconsumption. ➢ Realize the over-pumping of natural resources.
Content related anchor points	*Justification of concept unit's selection.* **Mathematical modeling** and **culturally relevant pedagogy** are both pedagogical approaches that rely on students' knowledge of everyday situations, yet mathematics education research has not fully attended to the ways in which they can be united in the classroom. In our case we propose to use an interpretation of culture as students' lived experiences of the natural environment, a perspective drawn from the funds of knowledge approach, which can uncover knowledge that is relevant for mathematical tasks and that can support socially conscious reflection for softening power relations and restoration of social justice. Five indicative activities denoting the modeling[1] process with tactile and digital tools. The actions are not independent of each other but followed with the aim of giving rise to discussions about the way in which nature works.

[1] More information about modelling can be found in the annex.

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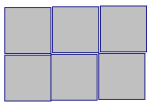
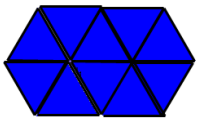
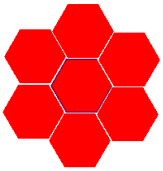
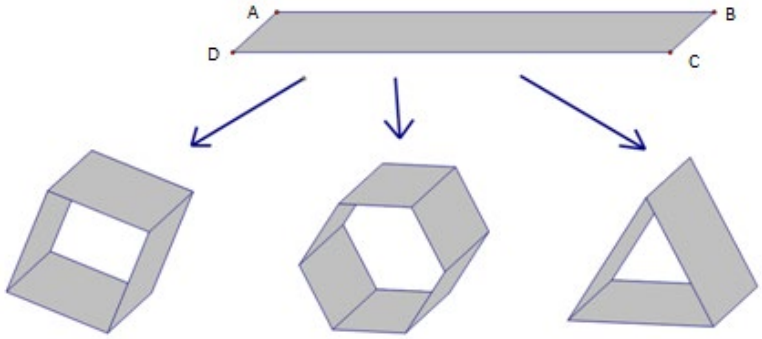
Methodology related anchor points	This unit include these five activities that are indicative. Depending on each classe's framework the teacher could decide to use all of them or part and also to expand the idea developing more activities. denoting the modeling[2] process with tactile and digital tools. The actions are not independent of each other but followed with the aim of giving rise to discussions about the way in which nature works. This concept proposes a pedagogical model, suggesting that the cycle of mathematical modeling provides key moments to access students' culturally based knowledge and that this approach can address weaknesses in typical implementations of culturally relevant pedagogy. Activity 1 This action discusses the issue of covering the plane with regular polygons. Students are asked to cover the level with equal squares.  Picture 1 Students are asked to cover the plane with equal equilateral triangles.  Picture2 Students are asked to cover the plane with equal hexagons.  Picture 3 The interpretation of why the only regular shapes that covert the plane are the square, the equilateral triangle as well as the hexagon is one of the basic questions that has value to be discussed during the 1st action. Activity 2 In this action, students are given equal rectangular strips of ABCD paper and are asked to construct as shown in figure 4: (a) the side surface of a rectangular square prism by dividing the paper strip into four equal sections. (b) the side surface of a rectangular hexagonal prism by dividing the paper strip into six equal sections. (c) the side surface of a rectangular, triangular, prism, by dividing the strip of paper into three equal sections.

[2] More information about modelling can be found in the annex.



Picture 6

The main objective of this action is for students to realize that with the use of the same building material-strips of paper, different type-volume storage spaces are constructed. The choice of paper is indicative. Teachers can suggest other shapes.

Activity 3

In this action, students are given appropriate material, for example rice, to fill the different storage spaces and are asked to think and decide which of the warehouses is the largest —which has the largest volume.

Activity 4

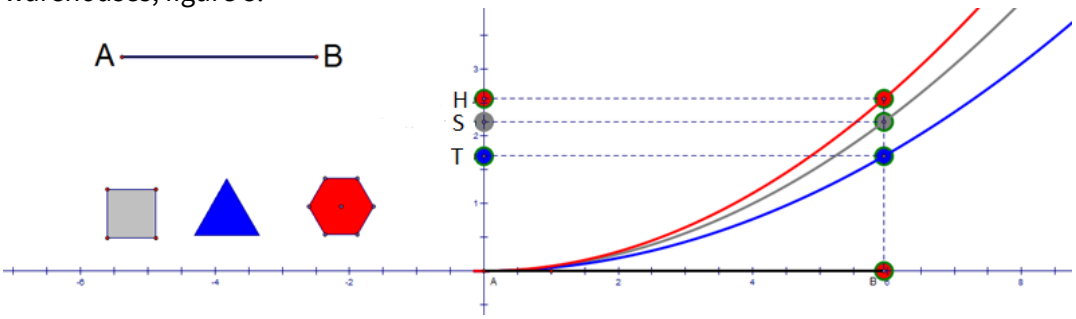
In the action, students are supported to confirm with the use of algebraic notation that the hexagonal prism is the largest storehouse while the triangular prism is the smallest.

Activity 5

In this action, students are given the opportunity through the mathematical modeling - see Annex- to proceed to the generalization of the arrangement of hexagon, square and triangle areas using dynamic geometry-Geogebra software. A snapshot shown in Figure 5. This file includes three dynamically interacting representations. The rectilinear section AB represents the building material whose length corresponds to the perimeters of the regular polygons-hexagon, square, triangle as well as the graphs corresponding to their areas.

When the length of AB is varying, it becomes apparent that the points of the graph of the area of the hexagon are always above the points of the graph of the square, while the points of the graph of the area of the triangle are always below the points of the graph of the square.

Through this activity, students are given the opportunity to conjecture on the shape of the beehive cells as well as to find out that the use of regular hexagonal cells satisfies the bees in relation to the surface coverage as well as the construction of optimal warehouses, figure 8.



Picture 7

H: Area of Hexagon, S: Area of Square, T: Area of Triangle

	 Picture 8
Language related anchor points	The concepts are formed in such a way to overcome language barriers and welcome students from different language backgrounds and cultures. The level of language demanded is basic and pictures are proposed to be used to reduce typical linguistic restrictions and increase students' participation.

Annex

The process of mathematical modeling is related to mathematization. The concept of mathematization has its origin in the *theory of Realistic Mathematics Education* (RME) and refers to activities of organizing and studying any kind of real or imaginary problem by mathematical means. It consists in translating the realistic problem into the symbolic mathematical world and vice versa, as well as reorganizing and reconstructing it in the world of mathematics. In the process of mathematization distinguishes horizontal and vertical folds (Treffers, 1987).

Horizontal mathematization: It concerns a transformative process of a realistic problem through an empirical approach, observation, or experimentation in a mathematical model. The activities that characterize horizontal mathematization usually include the visualization of the problem, the translation of physical and linguistic approaches into symbolic expressions in various ways as well as the discovery of relationships (De Lange, 1987).

Vertical mathematization: It concerns processes of conjecture, argumentation, or solution within the environment of the mathematical model and their interpretation from the point of view of the original problem. In addition, the generalization of the solution as well as further standardization can be included. Activities characteristic of vertical mathematization include the manipulation of the mathematical model, the focus on specific mathematical entities and relationships, integrating and combining different representations, modifying, or inventing new models.

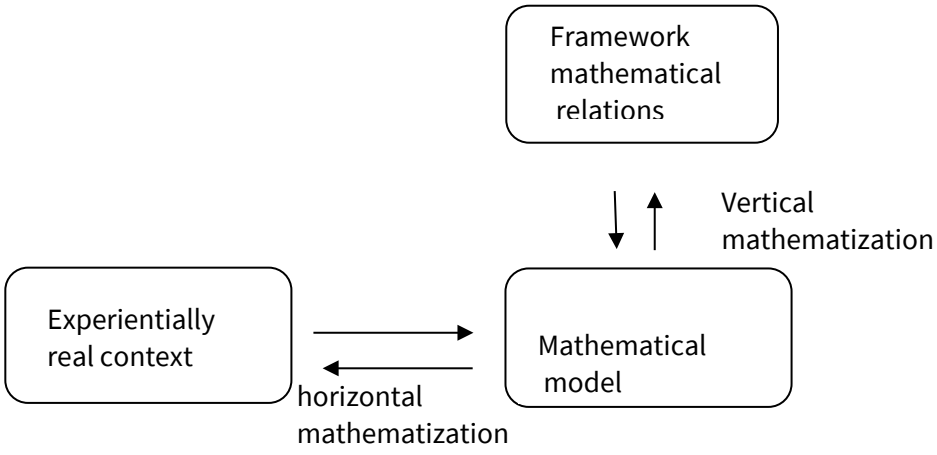


Figure 1: Horizontal and vertical mathematization (by Drijvers, 2003)

De Lange (1987) considers that in all phases of the activity of mathematization, both horizontal and vertical mathematization intertwine and complement each other, and that the horizontal aspect does not necessarily precede the vertical one. He emphasizes that the process of mathematization is personal and can take different paths depending on the students' perception of the realistic situation, their skills and solving abilities. According to De Lange (2006), the process of mathematization, which is performed by the student, has a circular character as shown in the figure 3.

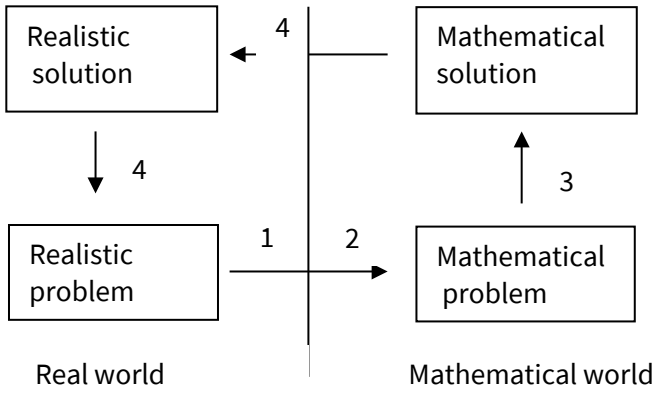


Figure 2: The cycle of mathematisation (by De Lange, 2006, p.17)

The figure marks the four main paths of mathematization.
(1): From an empirical real problem or situation, the student begins the process of understanding it by identifying the relevant mathematical concepts within it.
(2): Removal of irrelevant elements that actually exist and are not related to the problem.
(3): An attempt to speculate or solve the problem in the mathematical world.
(4): Attempt to interpret the mathematical solution from the point of view of the original, realistic situation.