



Unit Description

Title	Green Geometry 2 (Mathematics in the flower bed)
Age of students/grade	17-18 years
Length of the unit	1 x 50 minutes (up to 2 x 50 minutes)
Mathematical topic	Applications of differentiation and integration
Prerequisite knowledge	<ul style="list-style-type: none">• Setting up the equation of a polynomial function with given function values• Differentiation of polynomial functions• Integration of polynomial functions• Interpretation of the Integral as the area beneath the graph
Materials needed	<ul style="list-style-type: none">• Task sheet 1
Objectives	The student <ul style="list-style-type: none">• can apply knowledge about properties of polynomial functions in practical, real-life situations• can use polynomial functions to model real-life objects• can apply differentiation and integration in real-life contexts
Notes on work with migrant students	<ul style="list-style-type: none">• Students (particularly migrant students) can bring pictures from flower beds in gardens or parks in their own countries of origin

Lesson plan:**Lesson 1 (50 minutes)**

Time	Method	Content and comments
ca. 5 min	Exposition and teacher-student-communication	The teacher motivates the students for the topic of the lesson and explains its significance in everyday life.
ca. 35 min	Partner work, or work in small groups	The students solve the tasks on the worksheet in teams of two or three. The teacher goes round the class and helps with any problems that arise.
ca. 10 min	Teacher-student-communication	Comparison of results

Should the class be very engaged and interested in the tasks of lesson 1, students can work on an additional task (Task 4) either at the end of lesson 1 (time permitting), or in a second lesson.

The shape of the flower bed is described by a function f , where x is the width of the flower bed in metres and $f(x)$ is the height of the flowerbed in metres.

A gardener lays out a customer's flower bed in such a way that the flower bed starts at the edges of the garden and reaches its maximum height in the centre of the garden. The customer notes that the flowerbed should have a maximum 'height' of three metres. Furthermore, it is known that the function f is only defined for $0 \leq x \leq 10$.

- Room for your sketch:

- | Description in words | Mathematical description |
|---|---|
| The flower bed starts at the edges of the garden.
These edges are at $x = \underline{\hspace{2cm}}$ and $x = \underline{\hspace{2cm}}$. | I: <u> </u>
II: <u> </u> |
| The flower bed has its maximum height in the centre of the garden. The centre of the garden is at $x = \underline{\hspace{2cm}}$. | III: <u> </u> |
| The customer notes that the flower bed should have a maximum ‘height’ of three metres. | IV: <u> </u> |

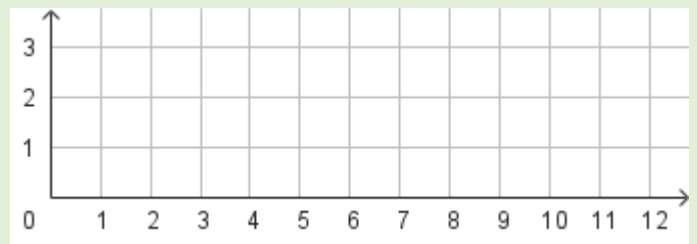
Polynomial function of degree ____: This is generally given by: $f(x) =$ _____

The polynomial function is given by: $f(x) =$ _____

Task 2:

Sketch the polynomial function that describes the flower bed again in the given coordinate system.

Estimate the actual area of the flowerbed using upper sums and lower sums (with 5 equal intervals).

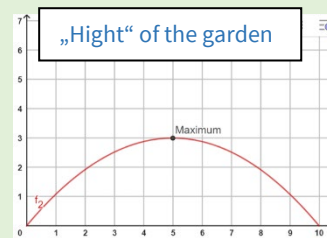
**Task 3:**

Determine the actual total area of the flower bed in the garden.

The shape of the flower bed is described by a function f , where x is the width of the flower bed in metres and $f(x)$ is the height of the flowerbed in metres.

A gardener lays out a customer's flower bed in such a way that the flower bed starts at the edges of the garden and reaches its maximum height in the centre of the garden. The customer notes that the flowerbed should have a maximum 'height' of three metres. Furthermore, it is known that the function f is only defined for $0 \leq x \leq 10$.

- Room for your sketch:



Width of the garden

- d) Using the above information, determine the term representation of a polynomial function f that describes the shape of the flower bed in the garden. Use the following properties to do this.

Description in words	Mathematical description
The flower bed starts at the edges of the garden. These edges are at $x = 0$ and $x = 10$.	I: $f(0) = 0$ II: $f(10) = 0$
The flower bed has its maximum height in the centre of the garden. The centre of the garden is at $x = 5$.	III: $f'(5) = 0$
The customer notes that the flower bed should have a maximum 'height' of three metres.	IV: $f(5) = 3$

Firstly, consider the degree of the polynomial function that is best and easiest for this flower bed.

Polynomial function of degree 2: This is generally given by: $f(x) = ax^2 + bx + c$

Find the polynomial function that describes the flower bed. Write a comprehensible description how you achieved your result in the space below.

$$f(x) = ax^2 + bx + c \quad \text{und} \quad f'(x) = 2ax + b$$

$$1: f(0) = 0 \quad a \cdot 0^2 + b \cdot 0 + c = 0 \quad \rightarrow c = 0$$

$$\text{II: } f(10) = 0 \quad a \cdot 10^2 + b \cdot 10 + c = 0 \rightarrow 100a + 10b = 0$$

$$\text{III: } f'(5) = 0 \quad 2 \cdot a \cdot 5 + b = 0 \quad \rightarrow 10a + b = 0$$

$$\text{IV: } f(5) = 3 \quad a \cdot 5^2 + b \cdot 5 + c = 3 \quad \rightarrow 25a + 5b = 3$$

From equation III and IV one gets the following system of equations:

$$\begin{array}{lcl} \text{III:} & 10a + b = 0 & | \cdot (-5) \\ \text{IV:} & 25a + 5b = 2 & | : (-25) \rightarrow a = -\frac{3}{25} \end{array}$$

$$IV: 25a + 5b = 3$$

$$III: \quad -50a - 5b = 0$$

$$\begin{array}{l} 10a + b = 0 \quad | -b \\ 10a = -b \end{array} \rightarrow b = \frac{30}{25} = \frac{6}{5}$$

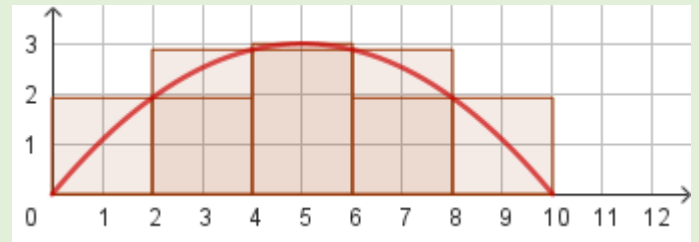
$$10a = -b$$

The polynomial function is given by $f(x) = -\frac{3}{25}x^2 + \frac{6}{5}x$

Task 2:

Sketch the polynomial function that describes the flower bed again in the given coordinate system.

Estimate the actual area of the flowerbed using upper sums and lower sums (with 5 equal intervals).



To estimate the area with the help of upper and lower sums, some function values are required, the values of which are given in the following table of values (rounded).

x	0	2	4	5	6	8	10
$f(x)$	0	1.92	2.88	3	2.88	1.92	0

Lower sum: $L_5 = 2 \cdot [f(0) + f(2) + f(4) + f(8) + f(10)]$
 $L_5 \approx 13.44$

Upper sum: $U_5 = 2 \cdot [f(2) + f(4) + f(5) + f(6) + f(8)]$
 $U_5 \approx 25.2$

One gets: $13.44 \text{ m}^2 < \text{actual area} < 25.2 \text{ m}^2$

Task 3:

Determine the actual total area of the flower bed in the garden.

$$\begin{aligned}
 A &= \int_0^{10} f(x) dx = \int_0^{10} \left(-\frac{3}{25}x^2 + \frac{6}{5}x \right) dx = \left(-\frac{3x^3}{25 \cdot 3} + \frac{6x^2}{5 \cdot 2} \right) \Big|_0^{10} \\
 &= \left(-\frac{3 \cdot 10^3}{25 \cdot 3} + \frac{6 \cdot 10^2}{5 \cdot 2} \right) - \left(-\frac{3 \cdot 0^3}{25 \cdot 3} + \frac{6 \cdot 0^2}{5 \cdot 2} \right) = (-40 + 60) = 20
 \end{aligned}$$

The actual total area of the flower bed is 20 m^2 .



Additional task



★ Task 4:

In order to make the best possible use of the garden area, the customer asks herself what the optimum width x the plot should have to maximise the total area of the flowerbeds in the garden. Determine the optimum width and describe how you did it in a comprehensible way.