



Unit Description

Title	Green Geometry
Age of students/grade	12 years
Length of the unit	8 x 50 minutes
Mathematical topic	Figures in the plane
Prerequisite knowledge	<ul style="list-style-type: none"> • Basic geometric figures • Perimeter and area of basic geometric figures
Materials needed	<ul style="list-style-type: none"> • Task sheet 1 • Task sheet 2 • Info sheet for the teacher • Various gardening tools and materials
Objectives	<p>The student</p> <ul style="list-style-type: none"> • can apply knowledge about properties of basic geometric figures (particularly triangle, rectangle, isosceles trapezoid, and circle) in practical, real-life situations • can apply the Pythagorean Theorem in real-life contexts • can use measurements and convert units in real-life contexts
Notes on work with migrant students	<ul style="list-style-type: none"> • Students (particularly migrant students) can bring pictures from gardens or parks in their own countries of origin
Comments	<ul style="list-style-type: none"> • The teacher must clarify with the municipality which green space in the city can be used by the students before the teaching units are realised. To avoid road traffic, it would be best to choose a green space in a park or playground. Above all, the location and size must be clarified, as well as the financial support from the municipality (costs incurred for e.g. soil, stones, gravel, bench, plants, etc.). The maintenance of the green space (especially watering) must also be clarified with the municipality. The harvesting of the selected vegetables (e.g. peas or pumpkin) should be left to the students. The teacher is responsible for ordering the flowers and plants as well as the required materials (e.g. gravel and stones) in co-operation with the local authority. • The teacher must look at the green space on-site in advance and take concrete measurements in order to plan the lessons. In the following suggested teaching units, a sample solution is worked out using a rectangular green space measuring 25 m x 8 m as an example. The actual size and design may vary greatly from the sample solution depending on the circumstances of the area and the ideas of the students.

Lesson plan:

Lesson 1 (50 minutes)

Time	Method	Content and comments
ca. 30 min	Partner work, then comparison in whole-class setting	<p>The lesson starts with a general overview of (selected) plane figures and equations for determining perimeter and area. (see task 1)</p> <p>Note: The pupils are already familiar with the plane figures mentioned, and perimeters and areas have already been calculated, but only with specific figures given in the textbook. The upcoming project is for practical application!</p> <p>Task 1 is solved to repeat the content already known.</p>
ca. 20 min	Exposition (explanation of project), Brainstorming, whole-class discussion	<p>The teacher presents pictures of gardens and introduces the students to the upcoming project. (See 'Presentation by the teacher')</p> <p>At the same time, the geometric figures mentioned above are taken into account. The teacher explains that the students will be given a rectangular green area in the city (as an example we use 25 m x 8 m). The students should already make suggestions for the design of this green space (separating and creating flower and vegetable beds, selecting plants, creating a footpath, possibly planning a bench, etc.). The students must be made aware that they should always incorporate the geometric figures just discussed (from task 1) when planning. Together in the plenary session, consider what equipment the students will need for the inspection in the following lesson (tape measure, paper and pencil for sketches, possibly a calculator and tablet).</p>

Lesson 2 (50 minutes)

Time	Method	Content and comments
50 min	Expedition, group work	<p>The students and the teacher inspect the chosen location on-site. The students first take the most important measurements together. (see Sketch 1 as an example). The dimensions may of course differ from the proposed sketch. Accordingly, they must take their own dimensions into account when making the calculations.</p> <p>The students are then divided into 2 groups:</p> <ul style="list-style-type: none">- Group 1 considers the planting (aesthetic preferences of the different countries of origin of the students should also be taken into account when selecting the planting)- Group 2 considers the detailed planning of the area (footpath, flower beds, required materials such as gravel, earth, crushed stone, stones)

Lesson 3 (50 minutes)

Time	Method	Content and comments
ca. 20 min	whole-class work	<p>The dimensions of the green space are drawn on a sheet of paper. Each student is to draw a sketch on their own sheet of paper, with one student taking it in turns to draw/add to the sketch on the board.</p>
ca. 30 min	whole-class work	<p>The ideas for designing the green space from the last lesson are then presented. The students can show the relevant plants or required materials on the beamer.</p>

Lesson 4 (50 minutes)

Time	Method	Content and comments
50 min	Whole class work, group work	<p>The sketch of the green space is now concretised and labelled with specific dimensions. (for an example, see Sketch 2)</p> <p>The students should also be made aware of other ways of measuring distances. To this end, the students are shown that distances in the United States are measured in miles, yards, feet and inches. The corresponding conversion table is first presented to the students and then the students are asked to convert the measurements of the measured space into different units.</p>

Lesson 5-6 (100 minutes)

Time	Method	Content and comments
100 min	Partner work, then comparison of results in whole-class setting	Calculations of area and perimeter in a real-life context; applications of Pythagoras' theorem; calculations of the number of pieces or quantity of materials required (see task 2)

Note: All the materials and tools are to be provided to the students before lessons 7-8 start!

Lesson 7-8 (100 minutes)

Time	Method	Content and comments
100 min	Group work	Realisation of all planned activities; Planting the beds, laying out the paths, ...

Task sheet 1

Task 1:

Fill in the gaps in the table correctly! Use the following equations.

Fill in the missing names and figures yourself!

$$A = a \cdot h_a = b \cdot h_b$$

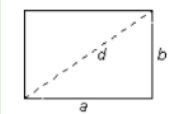
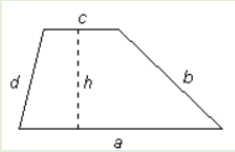
$$U = 2 \cdot (a + b)$$

$$A = a^2$$

$$A = \frac{(a+c) \cdot h}{2}$$

$$U = 2 \cdot r \cdot \pi$$

$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

Geometric figure	Sketch	Perimeter	Area
Rectangle			$A = a \cdot b$
		$U = 4 \cdot a$	
Parallelogram		$U = 2 \cdot (a + b)$	
Triangle		$U = a + b + c$	
		$U = a + b + c + d$	
Circle			$A = r^2 \cdot \pi$

Task sheet 1 – solutions

Task 1:

Fill in the gaps in the table correctly! Use the following equations.

Fill in the missing names and figures yourself!

$$A = a \cdot h_a = b \cdot h_b$$

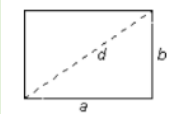
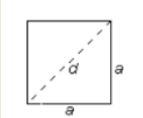
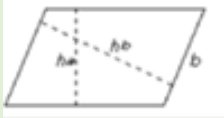
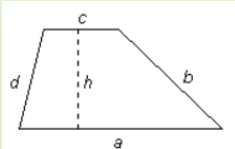
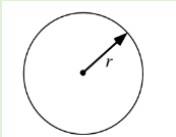
$$U = 2 \cdot (a + b)$$

$$A = a^2$$

$$A = \frac{(a+c) \cdot h}{2}$$

$$U = 2 \cdot r \cdot \pi$$

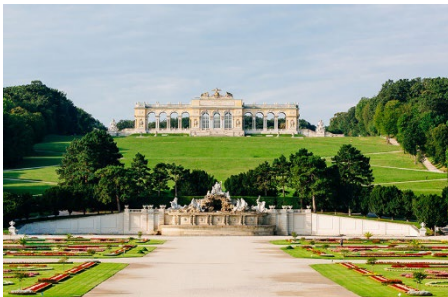
$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

Geometric figure	Sketch	Perimeter	Area
Rectangle		$U = 2 \cdot (a + b)$	$A = a \cdot b$
Square		$U = 4 \cdot a$	$A = a^2$
Parallelogram		$U = 2 \cdot (a + b)$	$A = a \cdot h_a = b \cdot h_b$
Triangle		$U = a + b + c$	$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$
Trapezoid		$U = a + b + c + d$	$A = \frac{(a + c) \cdot h}{2}$
Circle		$U = 2 \cdot r \cdot \pi$	$A = r^2 \cdot \pi$

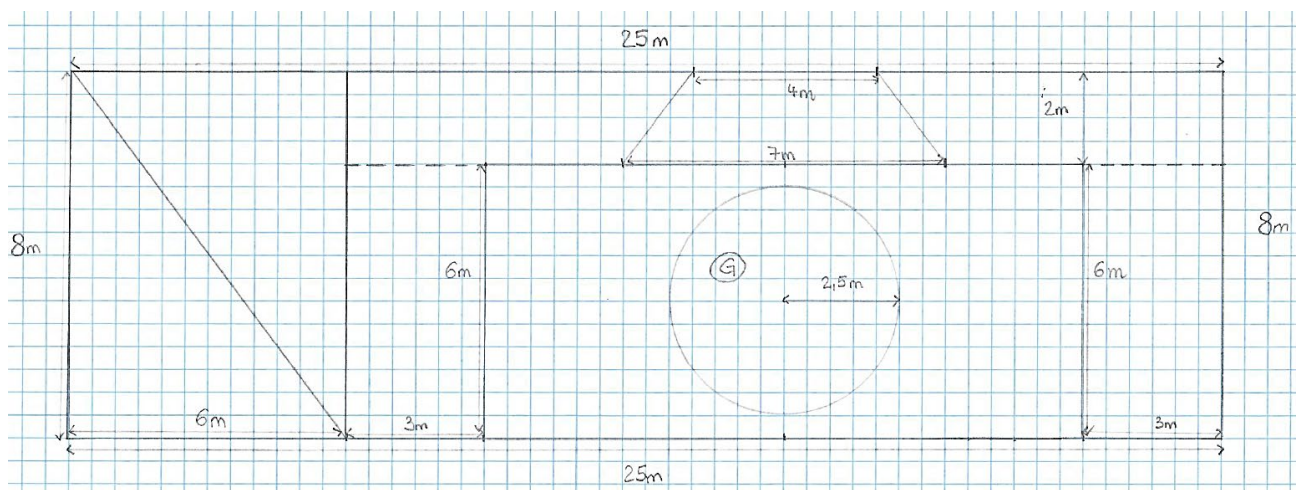
Info sheet for the teacher

Presentation by the teacher:

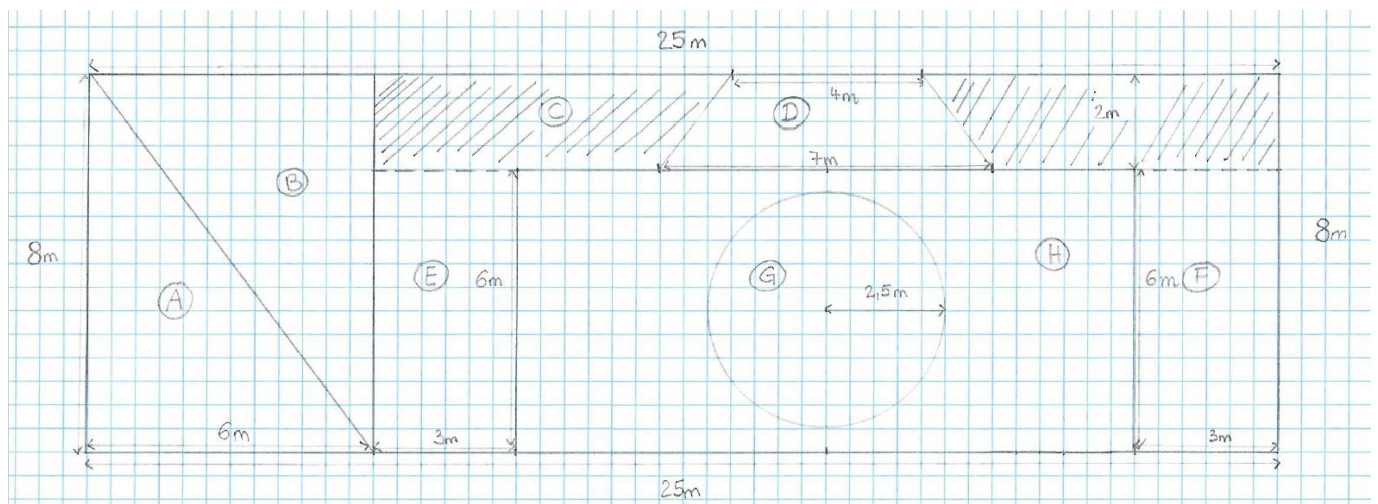
An example of a famous garden is the garden in front of Schönbrunn Palace and the Gloriette in Vienna.



Sketch 1



Sketch 2



Sketch 2 shows a detailed plan (see figure for corresponding dimensions):

- Areas A and B are triangles of equal area
- Area C is the hatched area (which is excluded by an isosceles trapezoid)
- Area D is an isosceles trapezoid
- Areas E and F are rectangles of equal area
- Area G is a circle
- Area H is a part of a rectangle, which is excluded from the circular area.

The selected green area has a size of 25 m x 8 m.

In the United States, distance measurements are done in the following units:

- *One mile is 1.609 km*
- *One yard is 91.44 cm*
- *One foot is 30.48 cm*
- *One inch is 2.54 cm*

Convert the following length measurements into the given units:

- $25\text{ m} = \underline{\hspace{1cm}}\text{ mile}$
- $25\text{ m} = \underline{\hspace{1cm}}\text{ foot}$
- $25\text{ m} = \underline{\hspace{1cm}}\text{ yard}$
- $25\text{ m} = \underline{\hspace{1cm}}\text{ inch}$

Solution:

- $25\text{ m} = 0.0155343\text{ miles}$
- $25\text{ m} = 82.021\text{ feet}$
- $25\text{ m} = 27.3403\text{ yard}$
- $25\text{ m} = 984.252\text{ inches}$

Task sheet 2

Task 2:

- a) Area A: Pumpkins are to be grown.



Calculate the corresponding surface area of the bed.

Think about a pumpkin variety and research on the internet how much space a pumpkin plant needs. How many plants can be planted?

Note: The pumpkin plant is chosen as an example here because it is very popular and well-known in Austria and can be grown in spring and harvested in autumn (so the summer holidays don't matter).

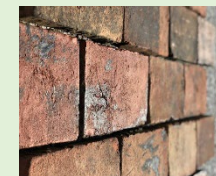
- b) Area B: Peas are to be grown.



You need 20-25 g of pea seed per square metre.

Calculate how many grams of seed you need for bed B!

- c) A path made of old bricks is to be laid between beds A and B.
Calculate the length of the path and give a way of laying the bricks.
Calculate the number of bricks required!



- d) Area D represents an isosceles trapezoid and is to be designed with gravel so that, for example, a bench can be placed there.
How many kg of gravel are needed if around 6kg of gravel are required for 1 m²?

- e) Area C is to be planted with a total of 4 hardy trees.
The entire area should then be filled with wood chips.

As a group, consider which hardy trees (maximum width of 3 metres) are suitable!
Calculate the size of the area!



- f) The areas E and F should be designed with hardy plants. Calculate that one plant requires approximately 120 cm x 120 cm of space.

Calculate how many plants can be planted.

Think about which plants can be used so that different (hardy) plants bloom continuously from March to October. The height of the plants should be approx. 50 to 150 cm.

Take into account the different popular and well-known plants in different countries! Create a table of the plants with growth height, flowering time and picture of the plant!

- g) The circular bed is to be planted with edible berries.

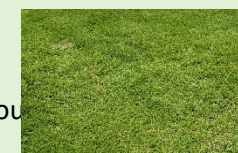
Calculate the area and select three types of berries to be planted.

The circular bed should also be surrounded with old bricks (dimensions 250 mm x 120 mm).

Calculate how many bricks are needed!



- h) Lawn is to be grown on the remaining area H.
Calculate the area and find out how much lawn seed is needed if 30 g per square metre is required!



- i) Finally, the sketch should be revised again and refined with specific colour representations.

Task sheet 2 – (examples of) solutions

Task 2:

- a) Area A: Pumpkins are to be grown.



Calculate the corresponding surface area of the bed.

Think about a pumpkin variety and research on the internet how much space a pumpkin plant needs. How many plants can be planted?

Note: The pumpkin plant is chosen as an example here because it is very popular and well-known in Austria and can be grown in spring and harvested in autumn (so the summer holidays don't matter).

$$A = \frac{6 \cdot 8}{2} = 24 \text{ m}^2$$

According to an Internet research, a Hokkaido-pumpkin-plant needs approx. 2.5 m^2 room.

$$24 / 2.5 = 9.6$$

One can grow up to 9 pumpkin plants.

Task 2:

- b) Area B: Peas are to be grown.



You need 20-25 g of pea seed per square metre.

Calculate how many grams of seed you need for bed B!

$$A = \frac{6 \cdot 8}{2} = 24 \text{ m}^2$$

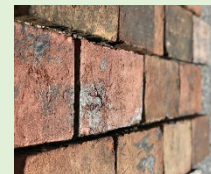
$$24 \cdot 20 = 480 \text{ g}$$

$$24 \cdot 25 = 600 \text{ g}$$

One needs between 480 g and 600 g of seeds.

Task 2:

- c) A path made of old bricks is to be laid between beds A and B.
Calculate the length of the path and give a way of laying the bricks.
Calculate the number of bricks required!



$$\text{Length of the path} = \sqrt{8^2 + 6^2} = 10$$

The path is 10 m long.

If a typical old brick measuring 250 mm x 120 mm is used, the bricks could be laid out lengthwise (2 rows in parallel would then be needed, so as the path not being too narrow).

$$\text{Per row one gets: } \frac{10 \text{ m}}{0.25 \text{ m}} = 40 \text{ pieces of brick}$$

For two rows, one would need 80 bricks.

Task 2:

- d) Area D represents an isosceles trapezoid and is to be designed with gravel so that, for example, a bench can be placed there.
How many kg of gravel are needed if around 6kg of gravel are required for 1 m²?

$$A = \frac{(7+4) \cdot 2}{2} = 11 \text{ m}^2; 11 \cdot 6 = 66 \text{ kg}$$

One needs approximately 66 kg of gravel.

Task 2:

- e) Area C is to be planted with a total of 4 hardy trees.
The entire area should then be filled with wood chips.

As a group, consider which hardy trees (maximum width of 3 metres) are suitable!
Calculate the size of the area!

Winter-hardy (narrow) small trees include, for example, the Photinia (*Photinia × fraseri*, left) or the harlequin willow (*Salix integra*, right).

$$A = 19 \cdot 2 - \frac{(7+4) \cdot 2}{2} = 27 \text{ m}^2$$



Task 2:

- f) The areas E and F should be designed with hardy plants. Calculate that one plant requires approximately 120 cm x 120 cm of space.

Calculate how many plants can be planted.

Think about which plants can be used so that different (hardy) plants bloom continuously from March to October. The height of the plants should be approx. 50 to 150 cm.

Take into account the different popular and well-known plants in different countries! Create a table of the plants with growth height, flowering time and picture of the plant!

$$A = 120 \cdot 120 = 14400 \text{ cm}^2 = 1.44 \text{ m}^2 \text{ per plant}; A_{\text{available}} = 3 \cdot 6 \cdot 2 = 36 \text{ m}^2; \frac{36}{1.44} = 25$$

One can plant approximately 25 plants.

Potential plants:

Plant	Growth height	Flowering time	Picture
Euphorbia (e.g. <i>Euphorbia epithymoides</i>)	Ca. 50 cm	April-July	
Spiraea (e.g. <i>Spiraea japonica</i>)	Ca. 50 cm - 150 cm	July-August	

Aster (e.g. <i>Symphyotrichum novae-angliae</i>)	Ca. 140 cm	September-October	
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Task 2:

- g) The circular bed is to be planted with edible berries.

Calculate the area and select three types of berries to be planted.
The circular bed should also be surrounded with old bricks
(dimensions 250 mm x 120 mm).
Calculate how many bricks are needed!



$$A = 2.5^2 \cdot \pi \approx 19.6 \text{ m}^2$$

Potential berries: raspberry, blackberry, blueberry.

$$U = 2 \cdot 2.5 \cdot \pi \approx 15.7 \text{ m}$$

$$\frac{15.7}{0.25} = 62.8$$

One needs approximately 63 bricks to surround the bed.



Task 2:

- h) Lawn is to be grown on the remaining area H.
Calculate the area and find out how much lawn seed
is needed if 30 g per square metre is required!



$$A = 13 \cdot 6 - 19.6 = 58.4 \text{ m}^2$$

$$58.4 \cdot 30 = 1752 \text{ g} \approx 1.8 \text{ kg}$$

One needs approximately 1.8 kg lawn seed.

Task 2:

- i) Finally, the sketch should be revised again and refined with specific colours and/or pictorial representations.

A new sketch may e.g. look like this:

